On Integrable Structures and Geometric Transitions in Supersymmetric Gauge Theories

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Based on 1303.4237 with Annamaria Sinkovics

An Exact Field Theoretic Correspondence was Proposed and proved few years ago (HYC, Dorey, Hollowood, Lee)

$$\text{Four Dimensional } \mathcal{N}=2 \text{ Gauge Theories + Nekrasov-Shatashvili Def } (\epsilon_1, \epsilon_2) = (\epsilon, 0) \xleftrightarrow{\text{CDHL}} \text{Two Dimensional } \mathcal{N}=(2,2) \text{ GLSM + twisted Chiral of mass } \epsilon$$

ϵ-deformed Seiberg-Witten Curves / Prepotential

1-loop exact twisted Superpotential

Quantum Integrable Systems

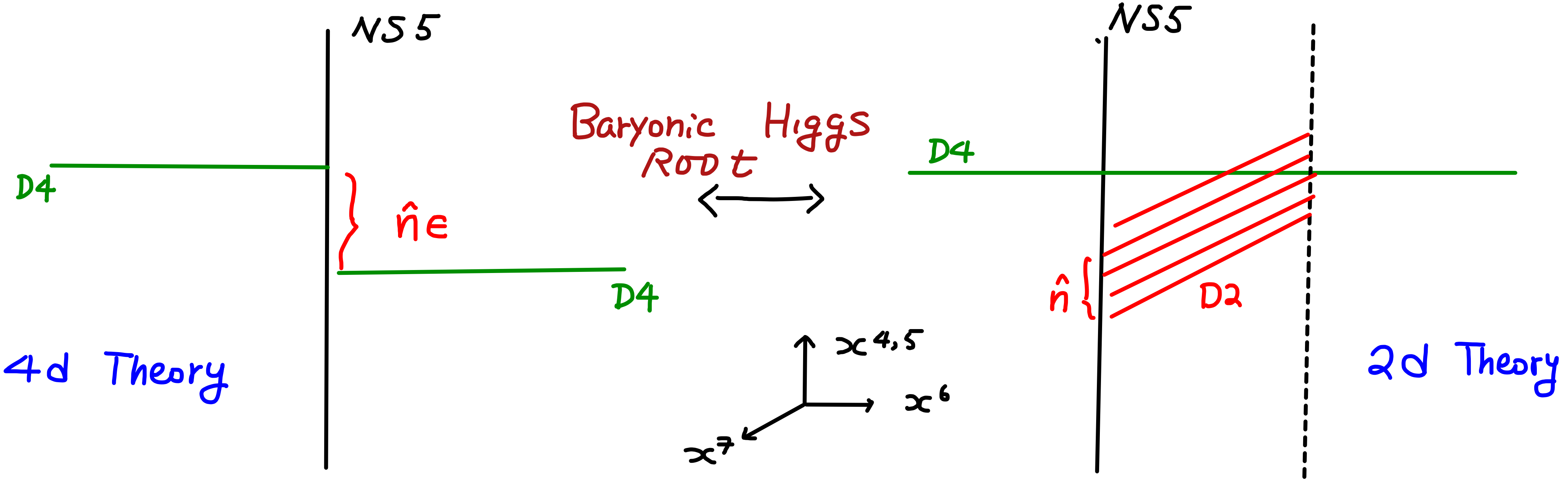
XXX Spin Chain, Calogero-Moser

Main Result

$$\frac{1}{\epsilon} \underbrace{\mathcal{F}_{Nek}(\vec{a}, \vec{m}, \epsilon)}_{\text{Baryonic Higgs Root}} = \underbrace{W(\vec{\lambda}, \vec{M})}_{\text{Vacua labeled by } \vec{n}} \quad (+\text{ Constant})$$

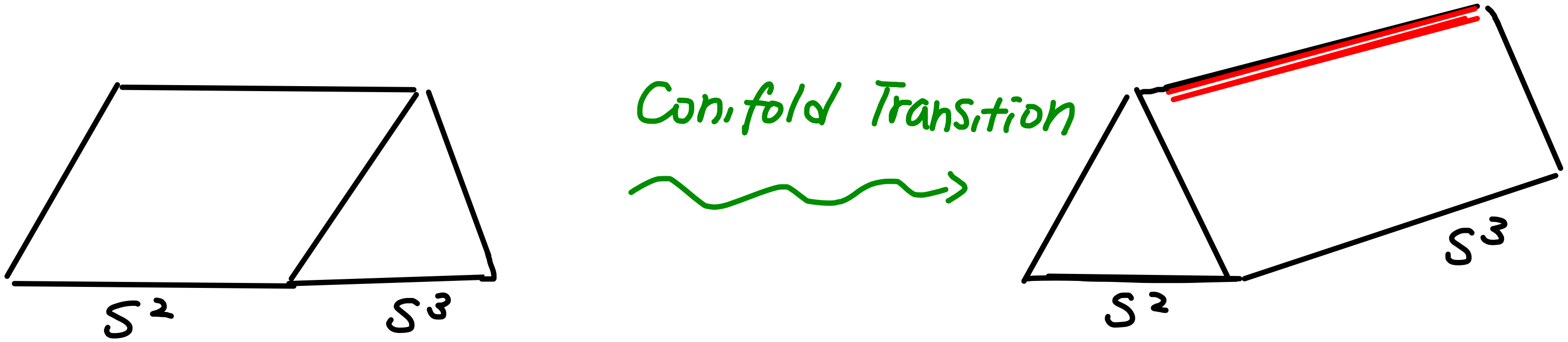
2d $\mathcal{N}=(2,2)$ GLSM is the World Volume Theory of "BPS Vortices" in 4d $\mathcal{N}=2$ Gauge Theories

D-Brane Pictures (Dorey-Hollowood-Lee)



Relating Intersecting Branes with Geometric Engineering (Karch-Smith)

Coulomb Branch = S^2 resolution , Higgs Branch = S^3 Deformation



Natural Place to study these issues is "Topological String"

Lifting theories on $\mathbb{R}^4_{\epsilon_{1,2}}$ to $S^1 \times \mathbb{R}^4_{\epsilon_{1,2}}$ (K-Theoretic Version)

$$Z^{K\text{-Th}}_{\text{Nek}}(\vec{a}, \vec{m}, \epsilon_{1,2}) = Z^{\text{Top}}_{\text{Closed}}(Q, t, q)$$

(Hollowood - Iqbal - Vafa, Awata - Kanno, Taki etc)

$$Z^{K\text{-Th}}_{\text{Vort}}(\vec{\lambda}, \vec{M}, \epsilon) = Z^{\text{Top}}_{\text{open}}(Q', t, q=1)$$

(Dimofte - Gukov - Holland)

$(\sim e^{W_{\text{1-loop}}})$

(i.e Toric Brane Insertions)

Geometric Transition

$$Z^{\text{Top}}_{\text{closed}} = Z^{\text{Top}}_{\text{open}}$$

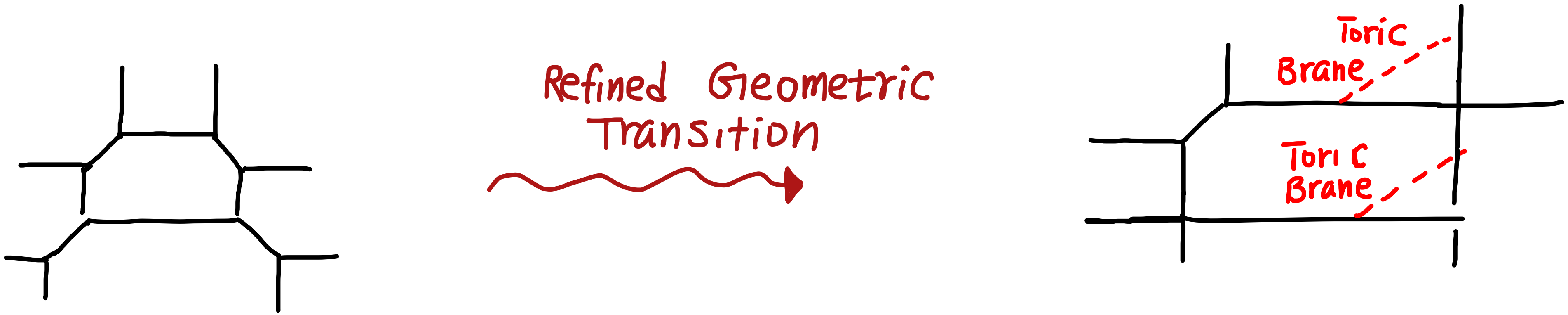
With appropriate identifications of parameters

In 1303.4237, we recast the "Truncated" $Z^{\text{Top}}_{\text{closed}}(Q, q, t)$
(HYC + Annamaria Sinkovics)

Truncation Condition $\qquad \vec{a} - \vec{m} = \vec{n} \in \qquad \vec{n} \in \mathbb{Z}$

= Baryonic Higgs Root

into $Z^{\text{Top}}_{\text{open}}(\alpha', q, t)$ in refined topological String (Explicit Refined open top string Computations)



- The Gauge Holonomy on the toric Branes is given by "MacDonald Polynomials" (Reduces to Schur Polynomial in unrefined limit)
- For multiple Stacks of Toric Branes, need to include "Refined Ooguri-Vafa Factor" to recover truncated $Z^{\text{Top}}_{\text{closed}}$