Unification of IIA IIB Supergravities

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- **Stringy unification of type IIA and IIB supergravities under N=2 D=10 supersymmetric double field theory**

with Kanghoon Lee, Jeong-Hyuck Park and Yoonji Suh   arXiv:1206.3478

Also bsed on   1011.1324, 1102.0419, 1105.6924, 1109.2035, 1112.0069, 1206.3478

Double Field Theory

O(D,D) T-duality manifest formulation of SUGRA, and beyond

[Siegel; Tseytlin; Hull, Zwiebach, Hohm]

Supersymmetric Double Field Theory (SDFT)

✓ **$\mathcal{N}=2$ $D=10$ SDFT unifies IIA and IIB supergravities (to the full order in fermions).**

✓ **Symmetries**:

- O(10,10) T-duality

cf. West E11

- Gauge symmetry
  - DFT-diffeomorphism (generalized Lie derivative)

    = diffeomorphism + B-field gauge symmetry
  - A pair of local Lorentz symmetries : $\mathbf{Spin}(1,9)_L \times \mathbf{Spin}(9,1)_R$
  - Local $\mathcal{N}=2$ SUSY with 32 supercharges.

✓ It is crucial to identify the 'correct' field variables.

Field contents of N=2 D=10 SDFT

- **Bosons**
  - **NS-NS sector** $\left\{ \begin{array}{ll} \textbf{DFT-dilaton:} & d \\ \textbf{DFT-vielbeins:} & V_{Ap}\,, \quad \bar{V}_{A\bar{p}} \end{array} \right.$
  - **R-R potential:** $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$
- **Fermions** (Majorana-Weyl)
  - **DFT-dilatinos:** ρ^{α} **,** $\rho'^{\bar{\alpha}}$
  - **Gravitinos:** $\psi^{\alpha}_{\bar{p}}$ **,** $\psi'^{\bar{\alpha}}_{p}$

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✓ RR- potential and fermions are all singlet under O(10,10).

✓ The usual IIA and IIB exchange follows only after diagonal gauge fixing of the local Lorentz symmetries.

Covariant Derivatives

- For each of the DFT gauge symmetry, we introduce a corresponding **connection** and set **"semi-covariant derivative" :**

$$\mathcal{D}_A = \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A$$

which is compatibile with the whole NS-NS sector,

$$\mathcal{D}_A d = 0\,, \qquad \mathcal{D}_A V_{Bp} = 0\,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = 0\,,$$

- We can determine the torsionless connection, uniquely,

$$\begin{aligned}\Gamma_{CAB} = {} & 2(P\partial_C P\bar{P})_{[AB]} + 2\big(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\big)\partial_D P_{EC} \\ & - \frac{4}{9}\big(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\big)\big(\partial_D d + (P\partial^E P\bar{P})_{[ED]}\big),\end{aligned}$$

where, $P_{AB} := V_A{}^p V_{Bp}$ $\bar{P}_{AB} := \bar{V}_A{}^{\bar{p}} \bar{V}_{B\bar{p}}$.

- Combined with the projections, it can be fully covariantized.

ex) $P_A{}^C \bar{P}_B{}^D \mathcal{D}_C T_D\,, \qquad P^{AB}\mathcal{D}_A T_B$ etc.

Curvature

- Define the **semi-covariant DFT-curvature**,

$$S_{ABCD} := \frac{1}{2}(R_{ABCD} + R_{CDAB} - \Gamma^{E}{}_{AB}\Gamma_{ECD})$$

- With the help of projections, it can produce fully covariant curvatures,

$$(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD}$$

R-R field strength

- We define R-R field strength

$$\mathcal{F} := \mathcal{D}_{+}\mathcal{C} = \gamma^{A}\mathcal{D}_{A}\mathcal{C} + \gamma^{(11)}\mathcal{D}_{A}\mathcal{C}\bar{\gamma}^{A}$$

where $\mathcal{D}_{+}$ is a **covariant nilpotent operator,** $\mathcal{D}_{+}^{2} = 0$,

corresponding to twisted K-theory O(D,D) covariant exterior derivative.

N=2 D=10 SDFT (full order completion)

$$\mathcal{L}_{\rm Type\,II} = e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} + \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) - i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$

$$+i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}^{\star}_{p}\rho - i\bar{\psi}^{\bar{p}}\mathcal{D}^{\star}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}^{\star}_{q}\psi_{\bar{p}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}'^{\star}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\mathcal{D}'^{\star}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}'^{\star}_{\bar{q}}\psi'_{p}\Big]\,.$$

- ✓ Higher order fermions are inside torsions.
- ✓ The usual 1.5 formalism works.
- ✓ The Lagrangian is pseudo: we impose self-duality relation:

$$\left(1-\gamma^{(D+1)}\right)\left(\mathcal{F} - i\tfrac{1}{2}\rho\bar{\rho}' + i\tfrac{1}{2}\gamma^{p}\psi_{\bar{q}}\bar{\psi}'_{p}\bar{\gamma}^{\bar{q}}\right) \equiv 0\,.$$

cf. Waldram et. al

N=2 D=10 SDFT (full order completion)

NS–NS sector geometrized

RR– field strength

$$\mathcal{L}_{\mathrm{Type\,II}} = e^{-2d}\Big[\underbrace{\tfrac{1}{8}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD}} + \underbrace{\tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}})} - i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$

$$+ i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}^{\star}_{p}\rho - i\bar{\psi}^{\bar{p}}\mathcal{D}^{\star}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}^{\star}_{q}\psi_{\bar{p}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}'^{\star}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\mathcal{D}'^{\star}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}'^{\star}_{\bar{q}}\psi'_{p}\Big]\,.$$

fermionic kinetic terms

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$$\left(1 - \gamma^{(D+1)}\right)\left(\mathcal{F} - i\tfrac{1}{2}\rho\bar{\rho}' + i\tfrac{1}{2}\gamma^{p}\psi_{\bar{q}}\bar{\psi}'_{p}\bar{\gamma}^{\bar{q}}\right) \equiv 0\,.$$

N=2 Local SUSY (full order completion)

$$\delta_\varepsilon d = -i\tfrac{1}{2}(\bar{\varepsilon}\rho + \bar{\varepsilon}'\rho')\,,$$

$$\delta_\varepsilon V_{Ap} = i\bar{V}_A{}^{\bar{q}}(\bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_p - \bar{\varepsilon}\gamma_p\psi_{\bar{q}})\,,$$

$$\delta_\varepsilon \bar{V}_{A\bar{p}} = iV_A{}^q(\bar{\varepsilon}\gamma_q\psi_{\bar{p}} - \bar{\varepsilon}'\bar{\gamma}_{\bar{p}}\psi'_q)\,,$$

$$\delta_\varepsilon \mathcal{C} = i\tfrac{1}{2}(\gamma^p\varepsilon\bar{\psi}'_p - \varepsilon\bar{\rho}' - \psi_{\bar{p}}\bar{\varepsilon}'\bar{\gamma}^{\bar{p}} + \rho\bar{\varepsilon}') + \mathcal{C}\delta_\varepsilon d - \tfrac{1}{2}(\bar{V}^A{}_{\bar{q}}\,\delta_\varepsilon V_{Ap})\gamma^{(d+1)}\gamma^p\mathcal{C}\bar{\gamma}^{\bar{q}}\,,$$

$$\delta_\varepsilon \rho = -\gamma^p\hat{\mathcal{D}}_p\varepsilon + i\tfrac{1}{2}\gamma^p\varepsilon\,\bar{\psi}'_p\rho' - i\gamma^p\psi^{\bar{q}}\bar{\varepsilon}'\bar{\gamma}_{\bar{q}}\psi'_p\,,$$

$$\delta_\varepsilon \rho' = -\bar{\gamma}^{\bar{p}}\hat{\mathcal{D}}'_{\bar{p}}\varepsilon' + i\tfrac{1}{2}\bar{\gamma}^{\bar{p}}\varepsilon'\,\bar{\psi}_{\bar{p}}\rho - i\bar{\gamma}^{\bar{q}}\psi'_p\bar{\varepsilon}\gamma^p\psi_{\bar{q}}\,,$$

$$\delta_\varepsilon \psi_{\bar{p}} = \hat{\mathcal{D}}_{\bar{p}}\varepsilon + (\mathcal{F} - i\tfrac{1}{2}\gamma^q\rho\,\bar{\psi}'_q + i\tfrac{1}{2}\psi^{\bar{q}}\,\bar{\rho}'\bar{\gamma}_{\bar{q}})\bar{\gamma}_{\bar{p}}\varepsilon' + i\tfrac{1}{4}\varepsilon\bar{\psi}_{\bar{p}}\rho + i\tfrac{1}{2}\psi_{\bar{p}}\bar{\varepsilon}\rho\,,$$

$$\delta_\varepsilon \psi'_p = \hat{\mathcal{D}}'_p\varepsilon' + (\bar{\mathcal{F}} - i\tfrac{1}{2}\bar{\gamma}^{\bar{q}}\rho'\bar{\psi}_{\bar{q}} + i\tfrac{1}{2}\psi'^q\bar{\rho}\gamma_q)\gamma_p\varepsilon + i\tfrac{1}{4}\varepsilon'\bar{\psi}'_p\rho' + i\tfrac{1}{2}\psi'_p\bar{\varepsilon}'\rho'\,.$$

Unification of IIA and IIB SUGRAs

✓ Thanks to the pair of local Lorentz symmetries, $\mathbf{Spin}(1,9)_L \times \mathbf{Spin}(9,1)_R$, there is no distinction of IIA and IIB:

While the theory is unique, one can show that solutions are twofold:

IIA and IIB solutions

Thank you.