Integrability of Marginally Deformed ABJ(M) theories

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ABJM theory

- ABJM theory is a *3d* Chern-Simons-matter theory with $\mathcal{N}=6$ supersymmetry. The gauge group is $U(N)*U(N)$ with Chern-Simons levels k and $-k$.
- The matter fields are four scalars, Y^I and four fermions Ψ_I in the bi-fundamental represen-tation of the gauge group.

Star product

- The real *β*-deformation introduce the following star product *[Lunin, Maldacena, 05]* *[Imeroni, 08]*:

$$f * g = e^{i\pi\beta(Q_1^f Q_2^g - Q_2^f Q_1^g)} f g \,,$$

- The charges Q^f_i of the matter fields are

	Y^1	Y^2	Y^3	Y^4
	$\Psi^{\dagger 1}$	$\Psi^{\dagger 2}$	$\Psi^{\dagger 3}$	$\Psi^{\dagger 4}$
$U(1)_1$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
$U(1)_2$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$

β-deformation

- Then we replace all the products in the Lagrangian by the above star product.

- The produced theory has *three dimensional* $\mathcal{N}$*=2* supersymmetry. *[Imeroni, 08]*

Anomalous dimensions

- We want to compute the anomalous dimension of

$$\mathcal{O}^{I_1 \cdots I_L}_{J_1 \cdots J_L} \equiv \mathrm{Tr}\left(Y^{I_1} Y^{\dagger}_{J_1} Y^{I_2} Y^{\dagger}_{J_2} \cdots Y^{I_L} Y^{\dagger}_{J_L}\right), \; L \geq 2.$$

at two-loop level in the planar limit.

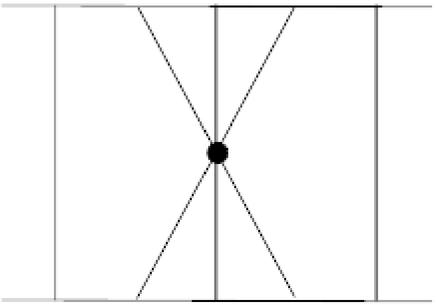
- *The undeformed case was studied in [Minaham, Zarembo, 08] [Bak, Rey, 08]*

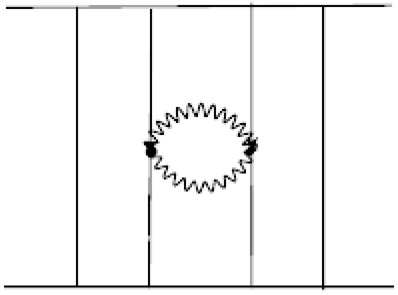
Spin chain

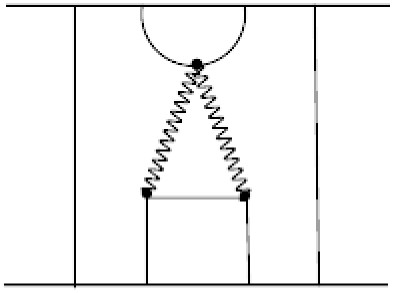
- The anomalous dimension matrix (ADM) can be expressed as a Hamiltonian acting on an alternating SU(4) spin chain.

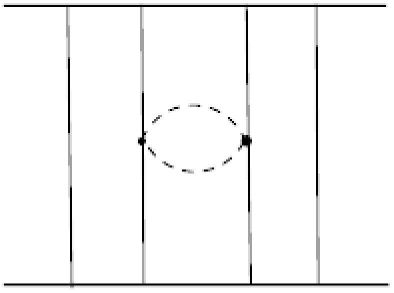
- The spins on the odd/even sides are in the fundamental/anti-fundamental representation of SU(4) R-symmetry group.

Contributing diagrams









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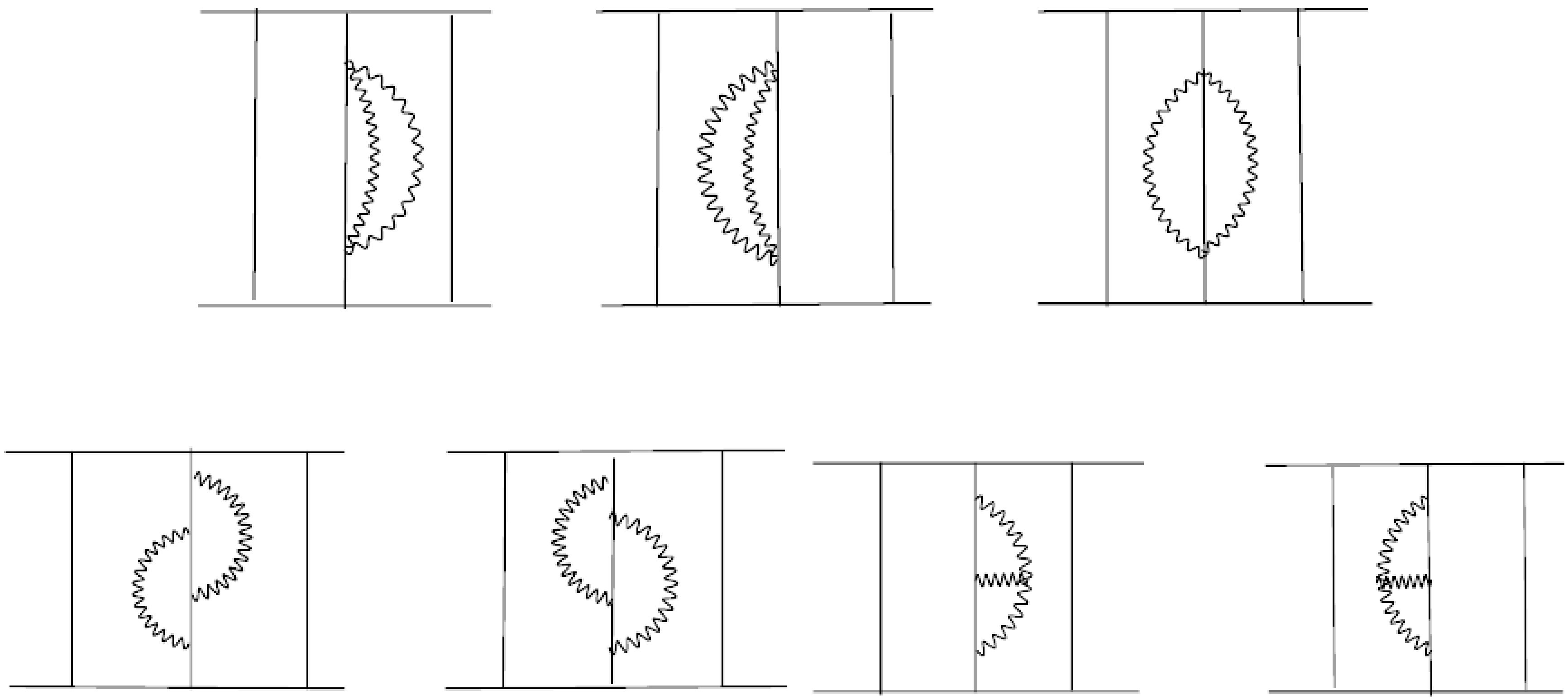

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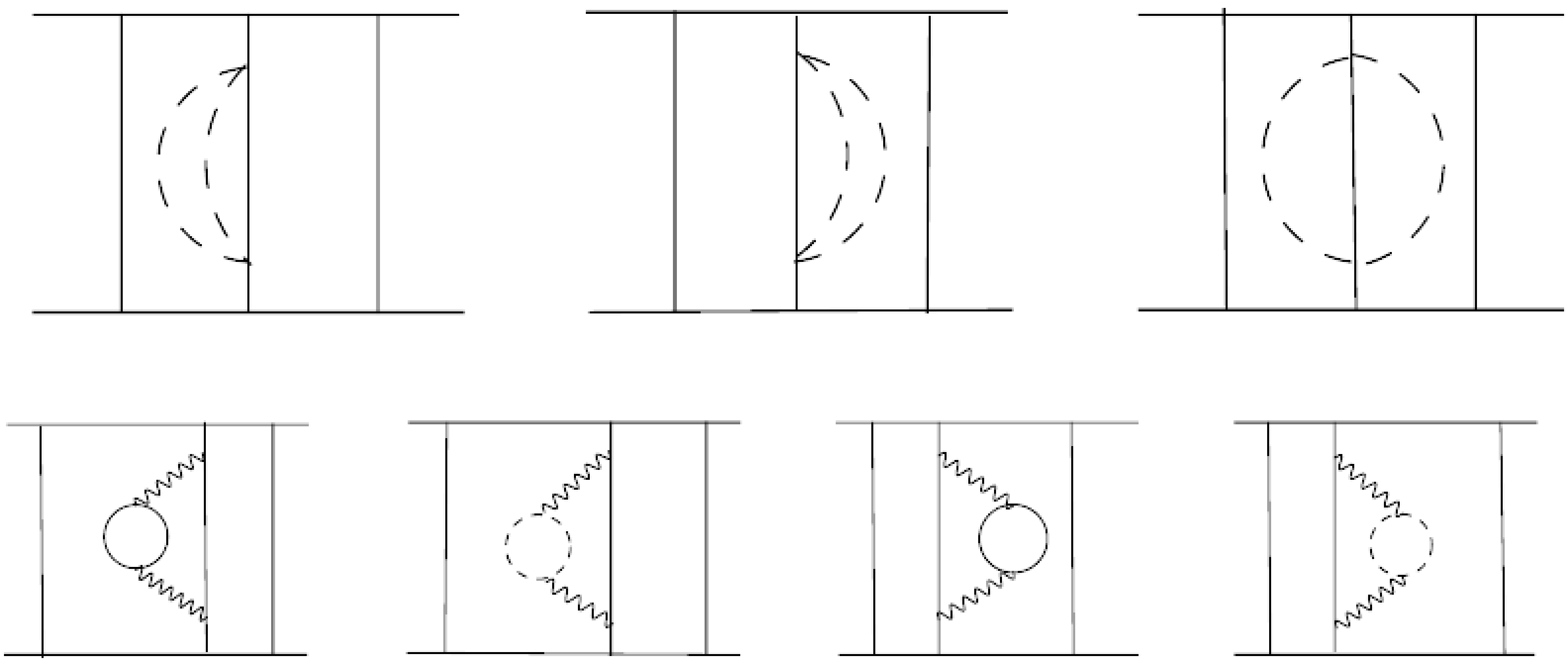


Two loop results

$$\widetilde{H}_{\rm total} = \widetilde{H}_B + H_F + H_{gauge} + H_Z$$
$$= \lambda^2 \sum_{i=1}^{2L} \left(\mathbb{I} - \widetilde{\mathbb{P}}_{i,i+2} + \frac{1}{2}\mathbb{P}_{i,i+2}\mathbb{K}_{i,i+1} + \frac{1}{2}\mathbb{P}_{i,i+2}\mathbb{K}_{i+1,i+2} \right).$$

$$\left(\widetilde{\mathbb{P}}_{i,i+2}\right)^{I_i I_{i+1} I_{i+2}}_{J_i J_{i+1} J_{i+2}} \equiv \exp(-i\pi\beta(Q^{J_i} \times Q^{J_{i+1}} + Q^{J_{i+1}} \times Q^{J_{i+2}} + Q^{J_{i+2}} \times Q^{J_i}))$$
$$\times \left(\mathbb{P}_{i,i+2}\right)^{I_i I_{i+1} I_{i+2}}_{J_i J_{i+1} J_{i+2}} .$$

$$Q^I \times Q^J \equiv \epsilon^{ij} Q^I_i Q^J_j$$

$$\mathbb{I}^{IJ}_{KL} = \delta^I_K \delta^J_L,\ \mathbb{P}^{IJ}_{KL} = \delta^I_L \delta^J_K,\ \mathbb{K}^{IJ}_{KL} = \delta^{IJ}\delta_{KL}.$$

R-matrices

$$\mathfrak{R}^{44}(u) = u\mathbb{I} + \mathbb{P},$$
$$\mathfrak{R}^{4\bar{4}}(u) = -(u+2)\mathbb{I} + \mathbb{K},$$
$$\mathfrak{R}^{\bar{4}4}(u) = -(u+2)\mathbb{I} + \mathbb{K},$$
$$\mathfrak{R}^{\bar{4}\bar{4}}(u) = u\mathbb{I} + \mathbb{P},$$

$$\widetilde{\mathfrak{R}^{44}}(u)^{IJ}_{KL} = \exp(-\frac{i}{2}\pi\beta(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{44}(u)^{IJ}_{KL},$$
$$\widetilde{\mathfrak{R}^{4\bar{4}}}(u)^{IJ}_{KL} = \exp(\frac{i}{2}\pi\beta(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{4\bar{4}}(u)^{IJ}_{KL},$$
$$\widetilde{\mathfrak{R}^{\bar{4}4}}(u)^{IJ}_{KL} = \exp(\frac{i}{2}\pi\beta(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\bar{4}4}(u)^{IJ}_{KL},$$
$$\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}(u)^{IJ}_{KL} = \exp(-\frac{i}{2}\pi\beta(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\bar{4}\bar{4}}(u)^{IJ}_{KL},$$

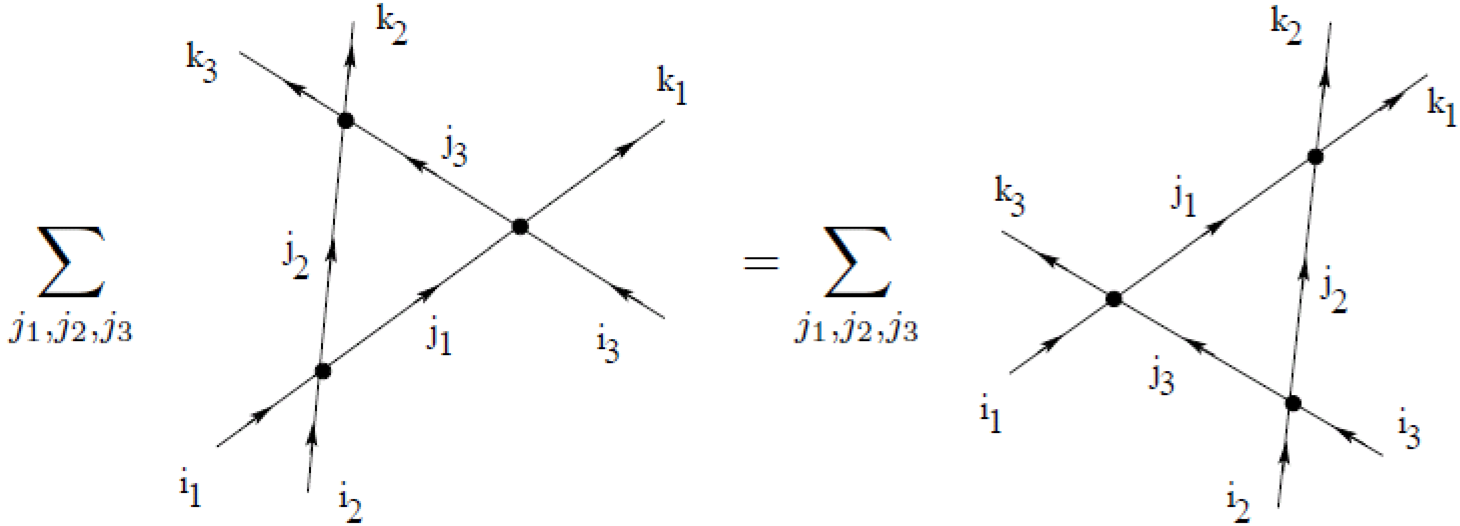
Yang-Baxter Equations

$$\widetilde{\mathfrak{R}^{44}}_{12}(u-v)\widetilde{\mathfrak{R}^{44}}_{13}(u)\widetilde{\mathfrak{R}^{44}}_{23}(v) = \widetilde{\mathfrak{R}^{44}}_{23}(v)\widetilde{\mathfrak{R}^{44}}_{13}(u)\widetilde{\mathfrak{R}^{44}}_{12}(u-v),$$

$$\widetilde{\mathfrak{R}^{44}}_{12}(u-v)\widetilde{\mathfrak{R}^{4\bar{4}}}_{13}(u)\widetilde{\mathfrak{R}^{4\bar{4}}}_{23}(v) = \widetilde{\mathfrak{R}^{4\bar{4}}}_{23}(v)\widetilde{\mathfrak{R}^{4\bar{4}}}_{13}(u)\widetilde{\mathfrak{R}^{44}}_{12}(u-v),$$

$$\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{12}(u-v)\widetilde{\mathfrak{R}^{\bar{4}4}}_{13}(u)\widetilde{\mathfrak{R}^{\bar{4}4}}_{23}(v) = \widetilde{\mathfrak{R}^{\bar{4}4}}_{23}(v)\widetilde{\mathfrak{R}^{\bar{4}4}}_{13}(u)\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{12}(u-v),$$

$$\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{12}(u-v)\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{13}(u)\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{23}(v) = \widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{23}(v)\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{13}(u)\widetilde{\mathfrak{R}^{\bar{4}\bar{4}}}_{12}(u-v).$$



Integrability

- Starting from these deformed R-matrices, we showed that ***this theory in the scalar sector at 2-loop level is intergrable in the planar limit.***

Bethe ansatz equations

- Using the algebraic Bethe ansatz method, we obtained the following asymptotic Bethe ansatz equations:

$$\exp(-\frac{i}{2}\pi\beta L - \frac{i}{2}\pi\beta N_m + i\pi\beta N_l)\left(\frac{l_a - \frac{i}{2}}{l_a + \frac{i}{2}}\right)^L = \prod_{a'\neq a}\frac{l_a - l_{a'} - i}{l_a - l_{a'} + i}\prod_{b=1}^{N_m}\frac{l_a - m_b + \frac{i}{2}}{l_a - m_b - \frac{i}{2}},$$

$$\exp(\frac{i}{2}\pi\beta L + \frac{i}{2}\pi\beta N_m - i\pi\beta N_l)\left(\frac{r_c - \frac{i}{2}}{r_c + \frac{i}{2}}\right)^L = \prod_{b=1}^{N_m}\frac{r_c - m_b + \frac{i}{2}}{r_c - m_b - \frac{i}{2}}\prod_{c'\neq c}\frac{r_c - r_{c'} - i}{r_c - r_{c'} + i},$$

$$\exp(\frac{i}{2}\pi\beta N_l - \frac{i}{2}\pi\beta N_r) = \prod_{a=1}^{N_l}\frac{m_b - l_a - \frac{i}{2}}{m_b - l_a + \frac{i}{2}}\prod_{b\neq b'}\frac{m_b - m_{b'} - i}{m_b - m_{b'} + i}\prod_{c=1}^{N_r}\frac{m_b - r_c + \frac{i}{2}}{m_b - r_c - \frac{i}{2}}.$$

Other results

- We also get similar integrable structure for the cases with non-supersymmetric γ-deformation and cases starting from ABJ theory.

THANKS FOR ATTENTIONS!