String theory landscape 2013

Superconformal Symmetry and Cosmological Attractors

Andrei Linde

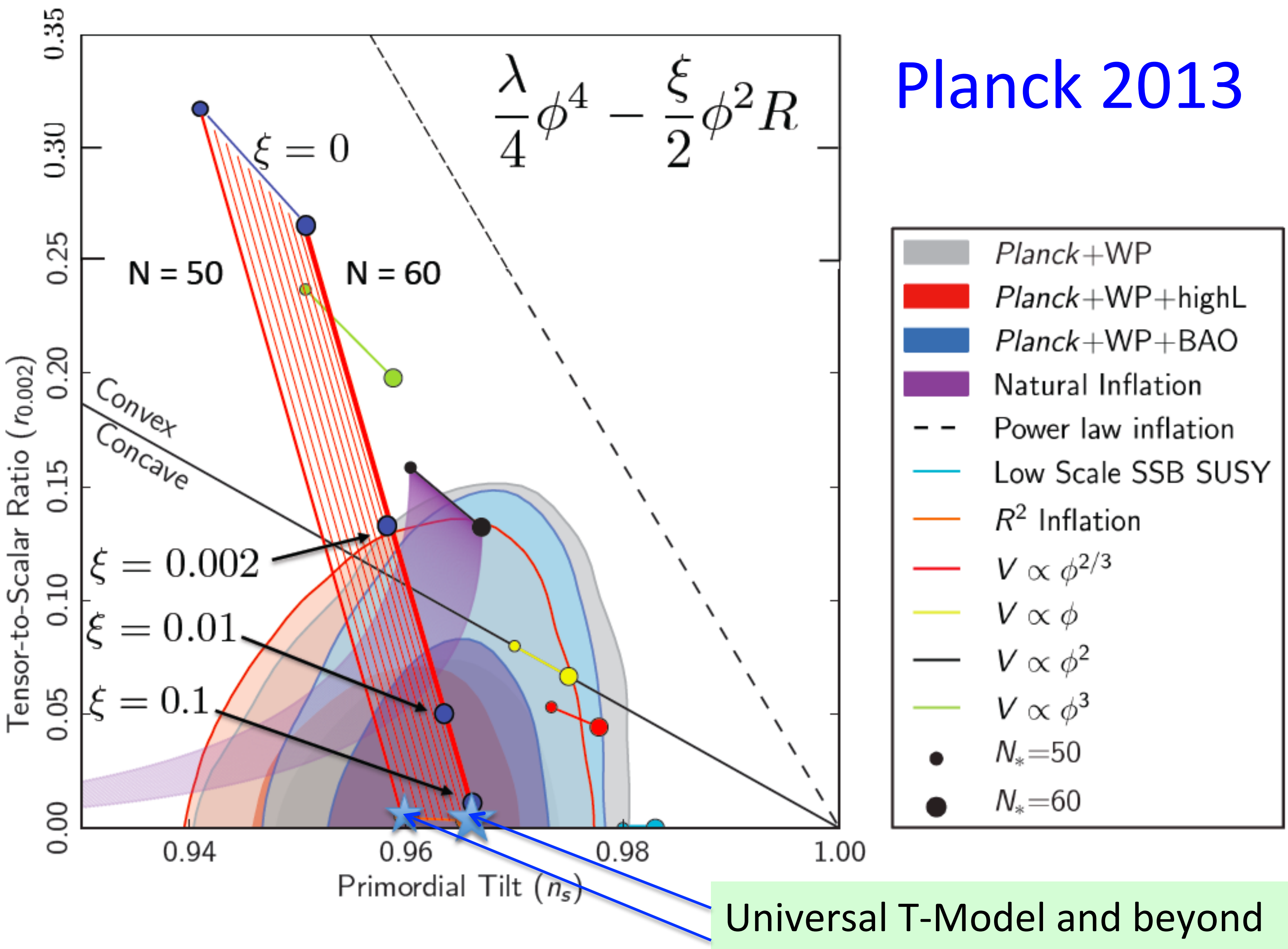
Based on: R. Kallosh, A.L. 1306.3211, 1306.3214, 1306.5220

We develop a new class of chaotic inflation models with spontaneously broken conformal or superconformal invariance. Observational consequences of a broad class of such models are stable with respect to strong deformations of the scalar potential.

In this class of models, inflation is possible even in the theories with very steep potentials because of their exponential flattening at the boundary of the moduli space. This effect can be described as inflation of moduli space, which exponentially stretches and flattens the inflaton potential.

In this sense, slow roll inflation **in** the landscape is facilitated by inflation **of** the landscape.

Planck 2013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$\xi = 0$
N = 50
N = 60
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Natural Inflation
Power law inflation
Low Scale SSB SUSY
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$N_* = 50$
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Universal T-Model and beyond

De Sitter from spontaneously broken conformal symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial_\nu\chi\, g^{\mu\nu} + \frac{\chi^2}{12}R(g) - \frac{\lambda}{4}\chi^4\right]$$

This theory is locally conformal invariant

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})}\mathrm{g}_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi$$

The field χ (x) is referred to as a conformal compensator, which we will call `**conformon**.' It has negative sign kinetic term, but this is not a problem because it can be removed from the theory by fixing the gauge symmetry, for example

$$\chi = \sqrt{6}$$

This gauge fixing can be interpreted as a spontaneous breaking of conformal invariance due to existence of a classical field $\chi = \sqrt{6}$

The action in this gauge:
dS or AdS

$$\mathcal{L} = \sqrt{-g}\left[\frac{R(g)}{2} - 9\lambda\right]$$

SPECIAL RELATIVITY REMINDER

$$\gamma \frac{v}{c} = \sinh \varphi$$

$$\gamma = \cosh \varphi$$

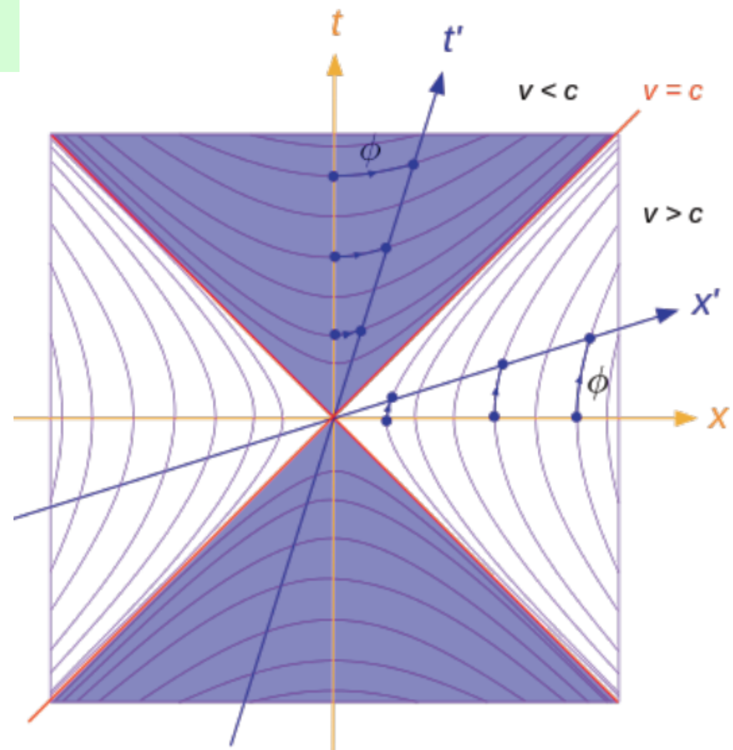
$$v = 0\,, \qquad \cosh \varphi = 1\,, \qquad \varphi = 0$$

$$v < c\,, \qquad \cosh \varphi > 1\,, \qquad |\varphi| > 0$$

$$v \to c\,, \qquad \cosh \varphi \to \infty\,, \qquad |\varphi| \to \infty$$

$$\tanh \varphi = \frac{v}{c}$$

φ is called rapidity



Space-time partition:
light-cone structure

$$ct' = ct \cosh \varphi - x \sinh \varphi$$

$$x' = -ct \sinh \varphi + x \cosh \varphi$$

The simplest conformally invariant two-field model of dS or AdS space and the SO(1,1) invariant conformal gauge

$$\mathcal{L} = \frac{\sqrt{-g}}{2}\left[(\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi) + \frac{\chi^2-\phi^2}{6}R(g) - \frac{(\phi^2-\chi^2)^2}{18}\right]$$

Local conformal symmetry

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})}\mathrm{g}_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi\,, \qquad \tilde{\phi} = \mathrm{e}^{\sigma(\mathrm{x})}\phi$$

The global SO(1,1) transformation is a boost between these two fields.

SO(1,1) invariant conformal gauge $\chi^2 - \phi^2 = 6$ Rapidity gauge

This gauge condition represents a hyperbola which can be parameterized by a canonically normalized field φ

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6}}\,, \qquad \phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6}}$$

The action in this gauge, dS/AdS

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - 9\lambda\right]$$

Chaotic inflation from conformal theory: T-Model

$$\mathcal{L} = \frac{\sqrt{-g}}{2}\left[(\partial_\mu\chi\partial^\mu\chi - \partial_\mu\phi\partial^\mu\phi) + \frac{\chi^2-\phi^2}{6}R(g) - \frac{(\phi^2-\chi^2)^2}{18}F\left(\phi/\chi\right)\right]$$

Here F is an arbitrary function of the ratio phi/chi. When this function is present, it breaks the SO(1,1) symmetry of the de Sitter model. Note that this is the only possibility to keep local conformal symmetry and to deform the SO(1,1) symmetry!

In rapidity gauge it becomes

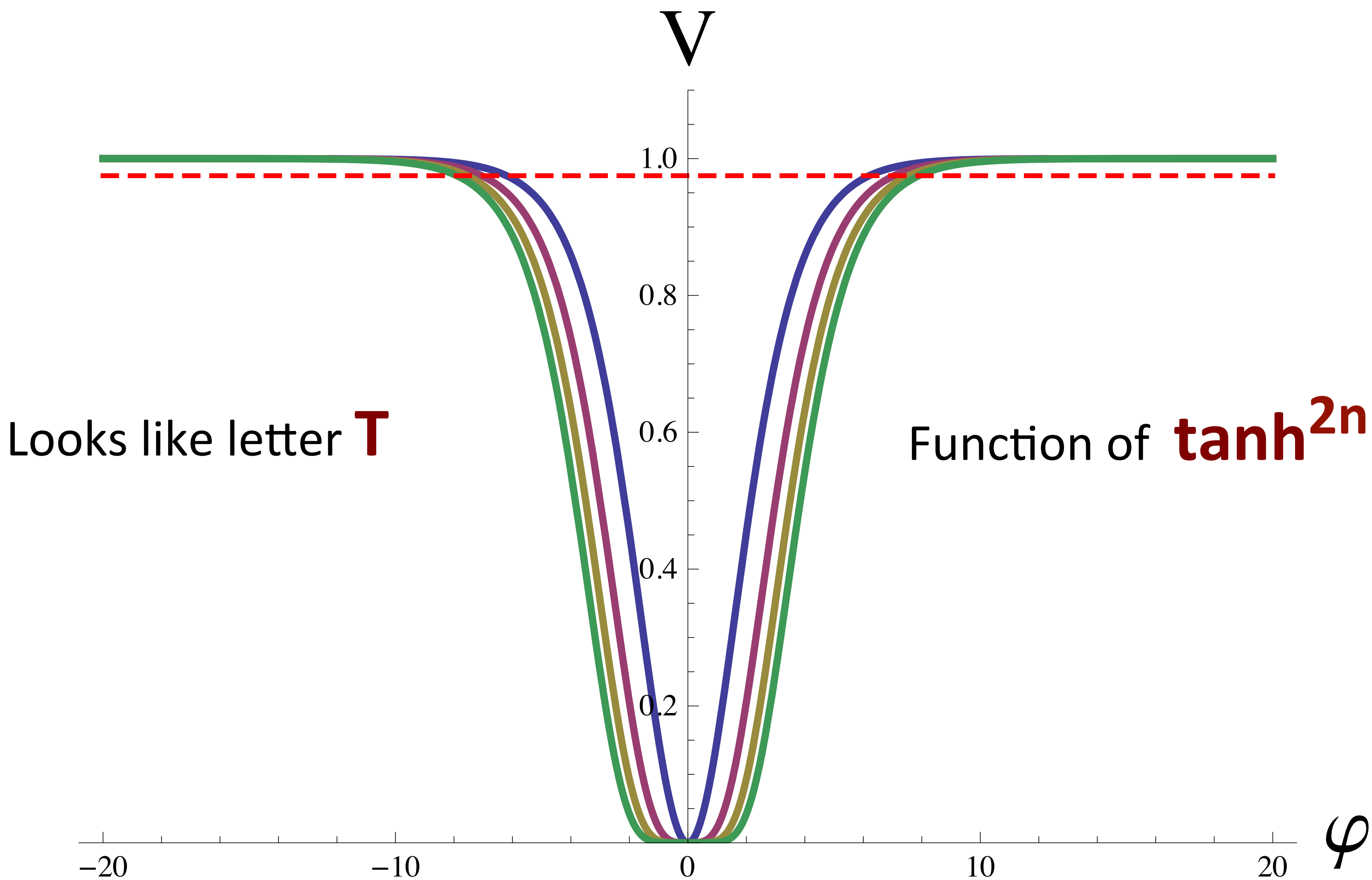
$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F(\tanh\varphi)\right]$$

The attractor behavior near a critical point where SO(1,1) symmetry is restored is the following: start with generic F(tanh), always get

$$n_s \approx 0.967 \qquad\qquad r \approx 0.0032$$

T-Model

$$F\left(\phi/\chi\right) = \lambda\left(\phi/\chi\right)^{2n} \qquad V(\varphi) = \lambda_n \, \tanh^{2n}(\varphi/\sqrt{6})$$



T-Model:

$$F\left(\phi/\chi\right) = \lambda\left(\phi/\chi\right)^{2n}$$

$$V(\varphi) = \lambda_n \tanh^{2n}(\varphi/\sqrt{6})$$

Functions $\tanh^{2n}(\varphi/\sqrt{6})$ are symmetric with respect to $\varphi \to -\varphi$, but to study inflationary regime in this model at $\varphi \gg 1$, it is convenient to represent them as follows:

$$V(\varphi) = \lambda\left(\frac{1 - e^{-\sqrt{2/3}\,\varphi}}{1 + e^{-\sqrt{2/3}\,\varphi}}\right)^{2n} = \lambda\left(1 - 4n\, e^{-\sqrt{2/3}\,\varphi} + O\left(n^2\, e^{-2\sqrt{2/3}\,\varphi}\right)\right)$$

The slow-roll equation for the field φ at $\varphi \gg 1$ in terms of the large e-folding number N is

$$\frac{d\varphi}{dN} = \frac{V'}{V} = 4n\sqrt{\frac{2}{3}}\, e^{-\sqrt{2/3}\,\varphi}\ .$$

For large N, this leads to a relation

$$e^{-\sqrt{2/3}\,\varphi(N)} = \frac{3}{8n\, N}\ .$$

Therefore for a given N, one has

$$\frac{V'}{V} = 4n\sqrt{\frac{2}{3}}\, e^{-\sqrt{2/3}\,\varphi(N)} = \frac{3}{2N}\sqrt{\frac{2}{3}}$$

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\Big(4n\sqrt{\frac{2}{3}}\, e^{-\sqrt{2/3}\,\varphi(N)}\Big)^2 = \frac{1}{2}\Big(\frac{3}{2N}\sqrt{\frac{2}{3}}\Big)^2$$

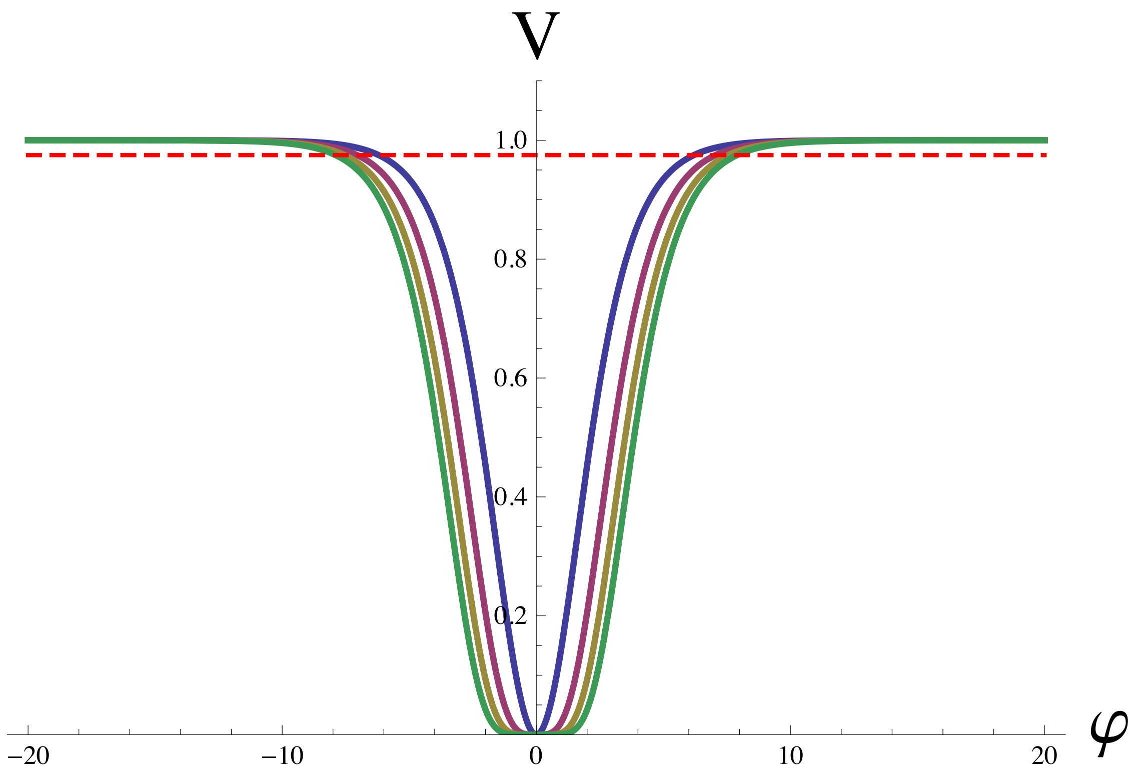
This result, in the leading order in $1/N$, is valid for any n. The same is true for the slow roll parameter η, and, consequently, for the parameters n_s and r: for the set of T-Models described in this section,

$$\boxed{1 - n_s = 2/N\,, \qquad r = 12/N^2}$$

ATTRACTOR

in the leading approximation in $1/N$

One could expect that these results may become increasingly unreliable for large n, but in fact the expansion parameter is $1/N$ for each model, so the difference of the predictions for $1 - n_s = 2/N$ and $r = 12/N^2$ in the slow-roll approximation is indeed $O(1/N^2)$. We checked this statement by explicitly comparing models $\tanh^2(\varphi/\sqrt{6})$ and $\tanh^{20}(\varphi/\sqrt{6})$, and we found, numerically, that in the slow-roll approximation in both cases $n_s \approx 0.967$ and $r \approx 0.0032$ for $N \approx 60$.



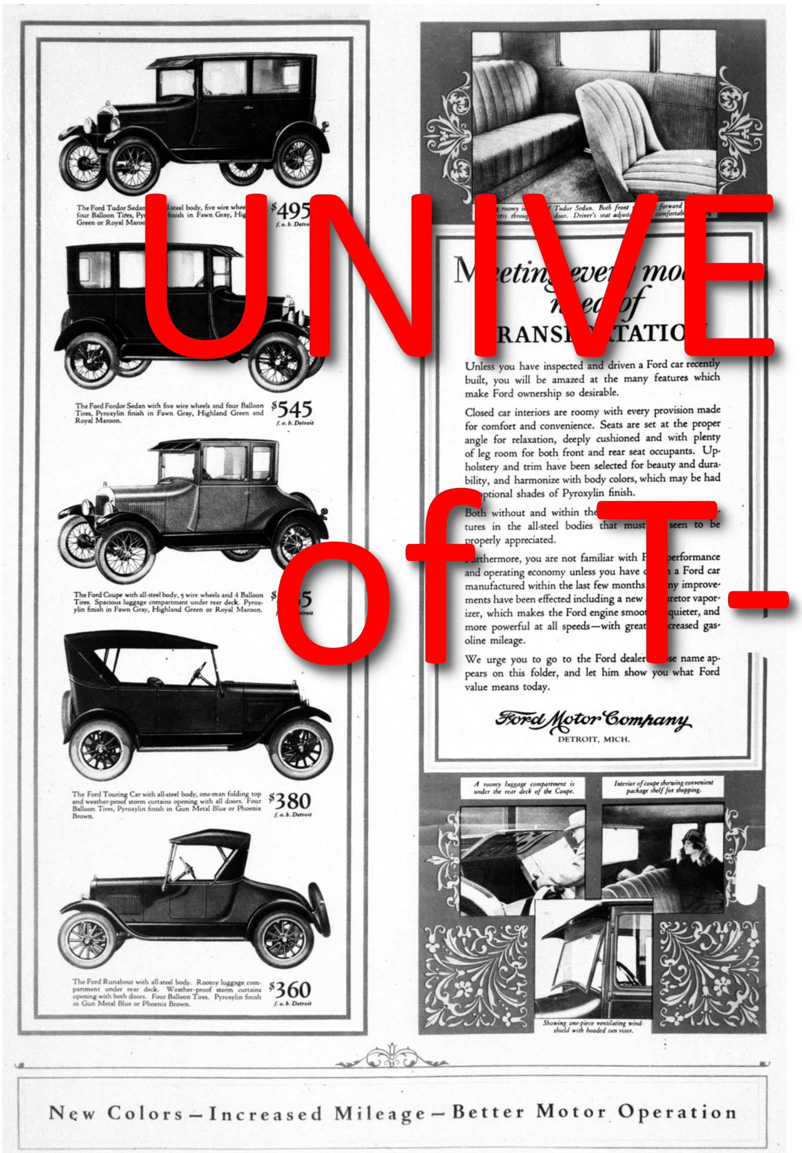
T-Model

Figure 1: Potentials for the T-Model inflation $\tanh^{2n}(\varphi/\sqrt{6})$ for $n = 1, 2, 3, 4$ (blue, red, brown and green, corresponding to increasingly wider potentials). We took $\lambda_n = 1$ for each of the potential for convenience of comparison. As we see, these potentials differ from each other quite considerably, especially at $\varphi \lesssim 1$: at small ϕ they behave as φ^{2n}. Nevertheless all of these models predict the same values $n_s = 1 - 2/N$, $r = 12/N^2$, in the leading approximation in $1/N$, where $N \sim 60$ is the number of e-foldings. The points where each of these potentials cross the red dashed line $V = 1 - 3/2N = 0.96$ correspond to the points where the perturbations are produced in these models on scale corresponding to $N = 60$.

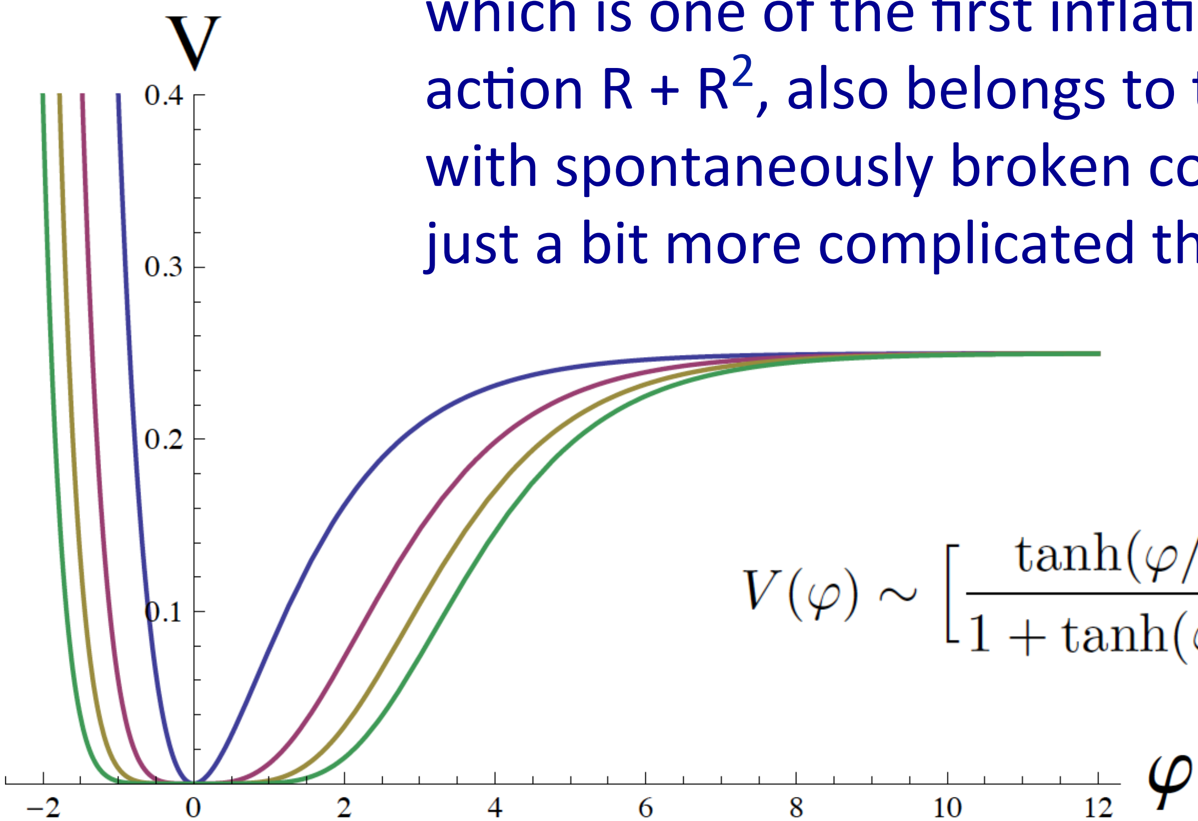
Any customer can have a car painted any color that he wants so long as it is black

Henry Ford

UNIVERSALITY of T-Model



To our surprise, we found that the Starobinsky model, which is one of the first inflationary models, with the action R + R^2, also belongs to this broad class of models with spontaneously broken conformal symmetry. It is just a bit more complicated than the basic T-Model



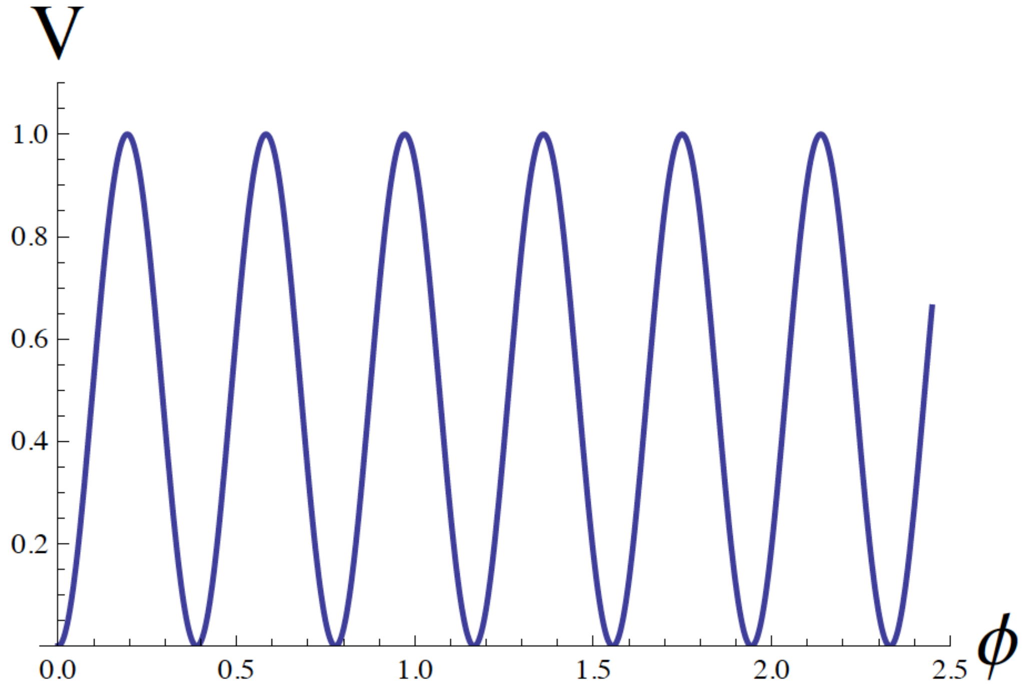
$$V(\varphi) \sim \left[\frac{\tanh(\varphi/\sqrt{6}))}{1+\tanh(\varphi/\sqrt{6}))}\right]^2 \sim \left(1 - e^{-\sqrt{2/3}\,\varphi}\right)^2$$

Figure 2: Models of conformal inflation based on generalizations of the Starobinsky model, with $F(\phi/\chi) \sim \frac{\phi^{2n}}{\chi^{2n-2}(\phi+\chi)^2}$, $n = 1, 2, 3, 4$.

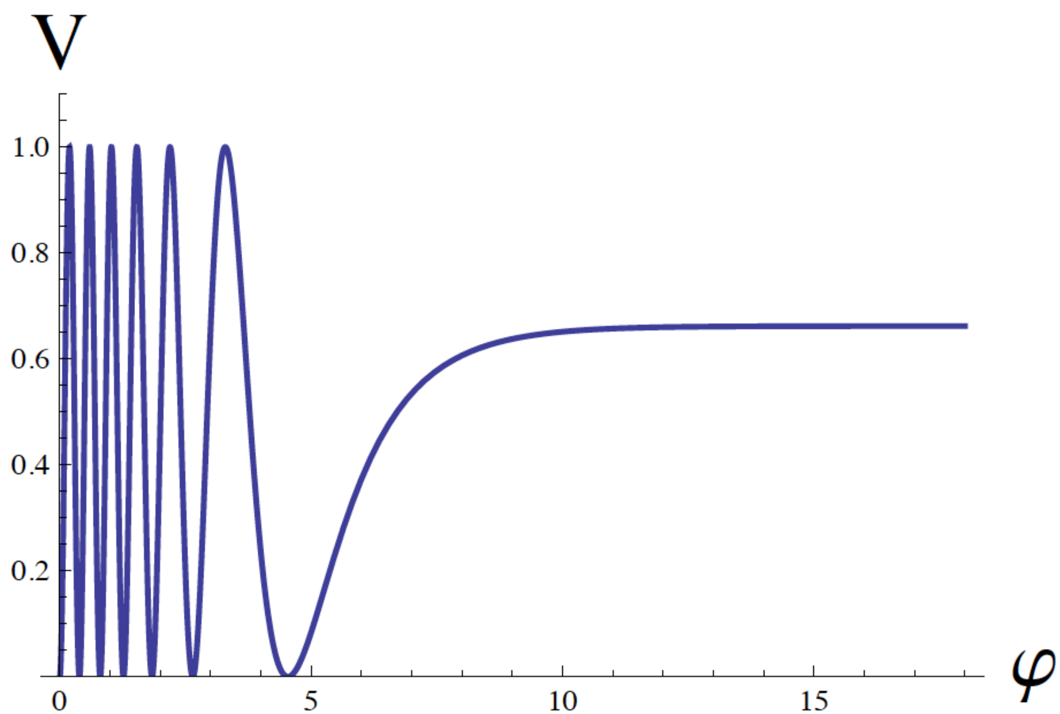
n=1

GENERALIZATIONS

Original potential



Potential in terms of the canonical field φ



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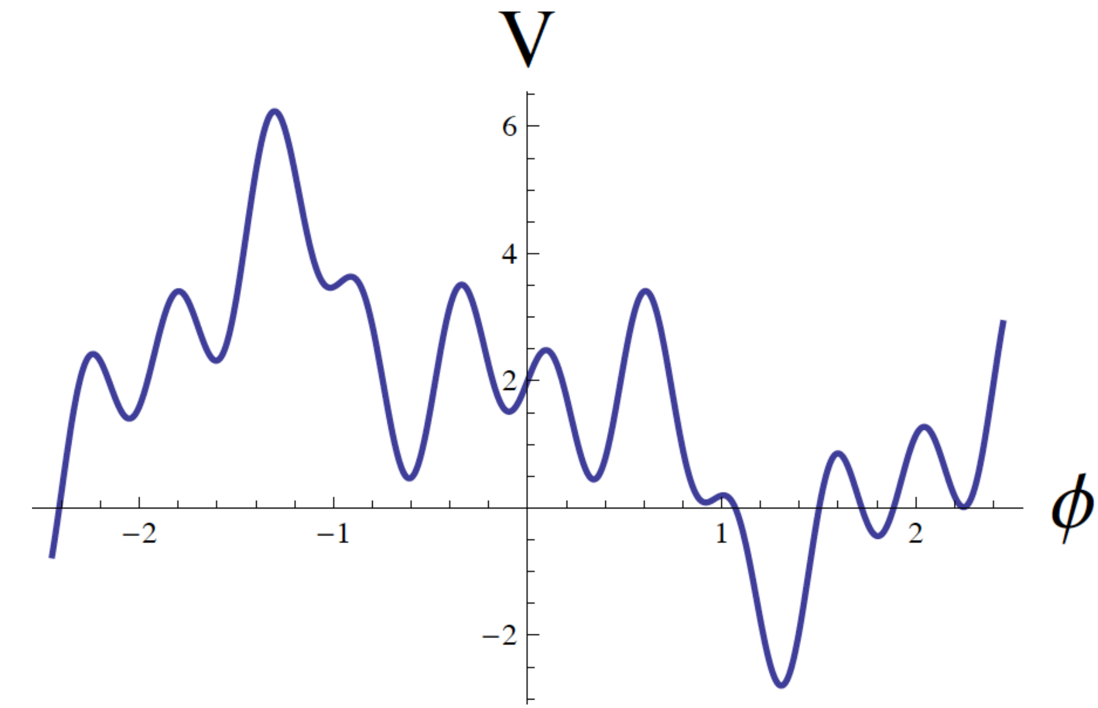

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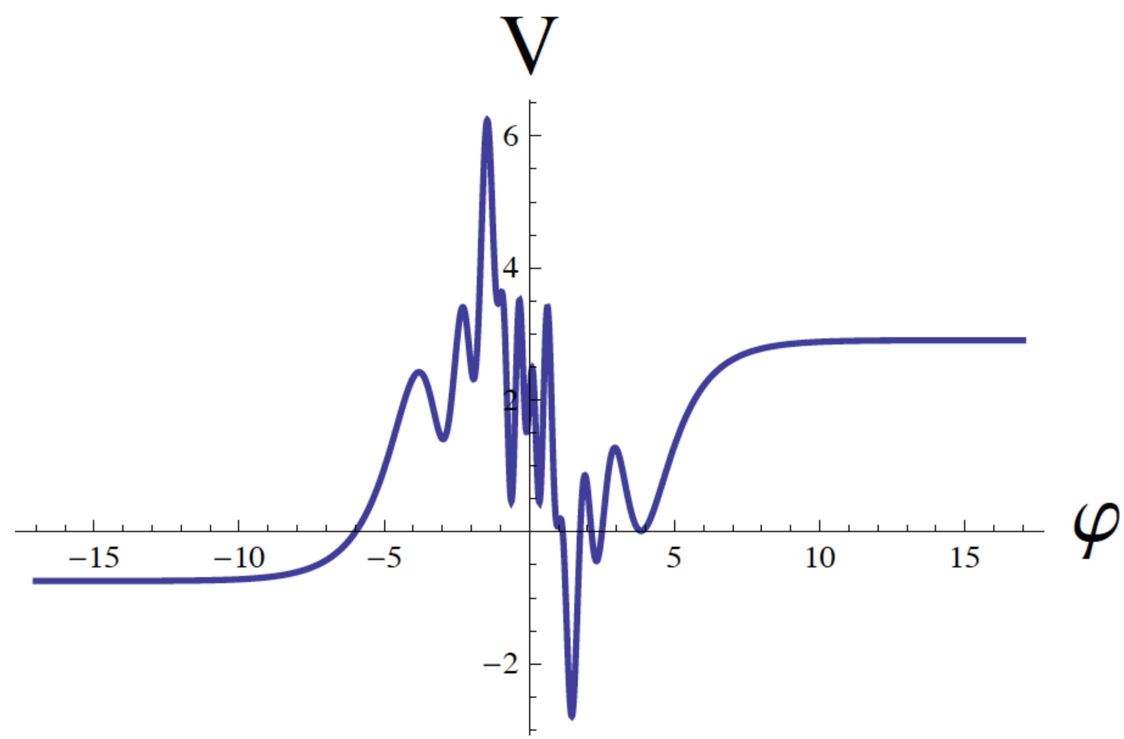
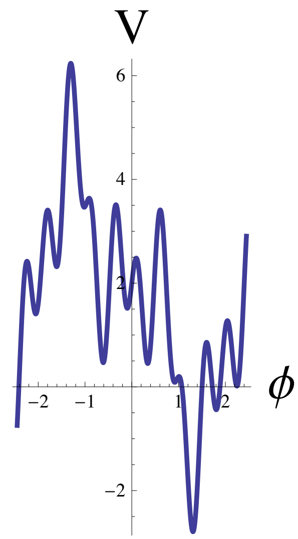
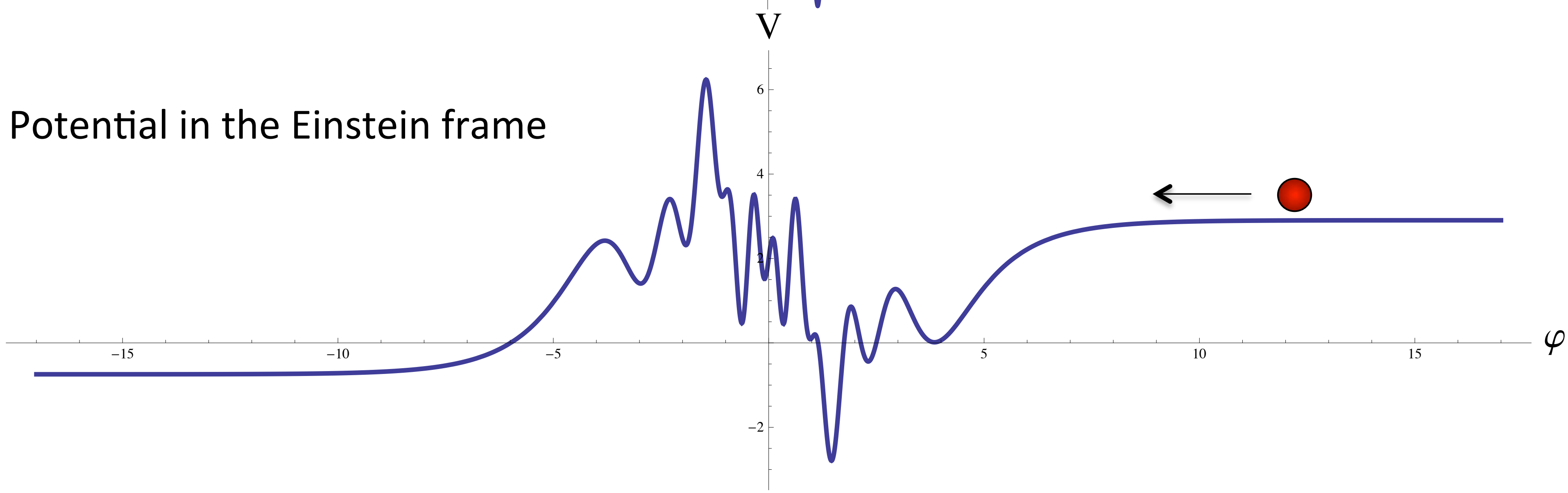


Stretching and flattening of the potential is similar to stretching of inhomogeneities during inflation

Potential in the original variables of the conformal theory



Potential in the Einstein frame



Inflation **in** the landscape is facilitated by inflation **of** the landscape

Universality of conformal inflation

Distance from the boundary of the moduli space

$$x = \sqrt{6} - \phi = \sqrt{6}(1 - \tanh \frac{\varphi}{\sqrt{6}}) \approx 2\sqrt{6}\ e^{-\sqrt{2/3}\,\varphi}$$

Expanding the potential in **X**:

$$V(\phi) = V(\sqrt{6})\,(1 - \sum c_n x^n) = V(\sqrt{6})\,(1 - \sum c_n \left(2\sqrt{6}\ e^{-\sqrt{2/3}\,\varphi}\right)^n)$$

Take the first term in the expansion:

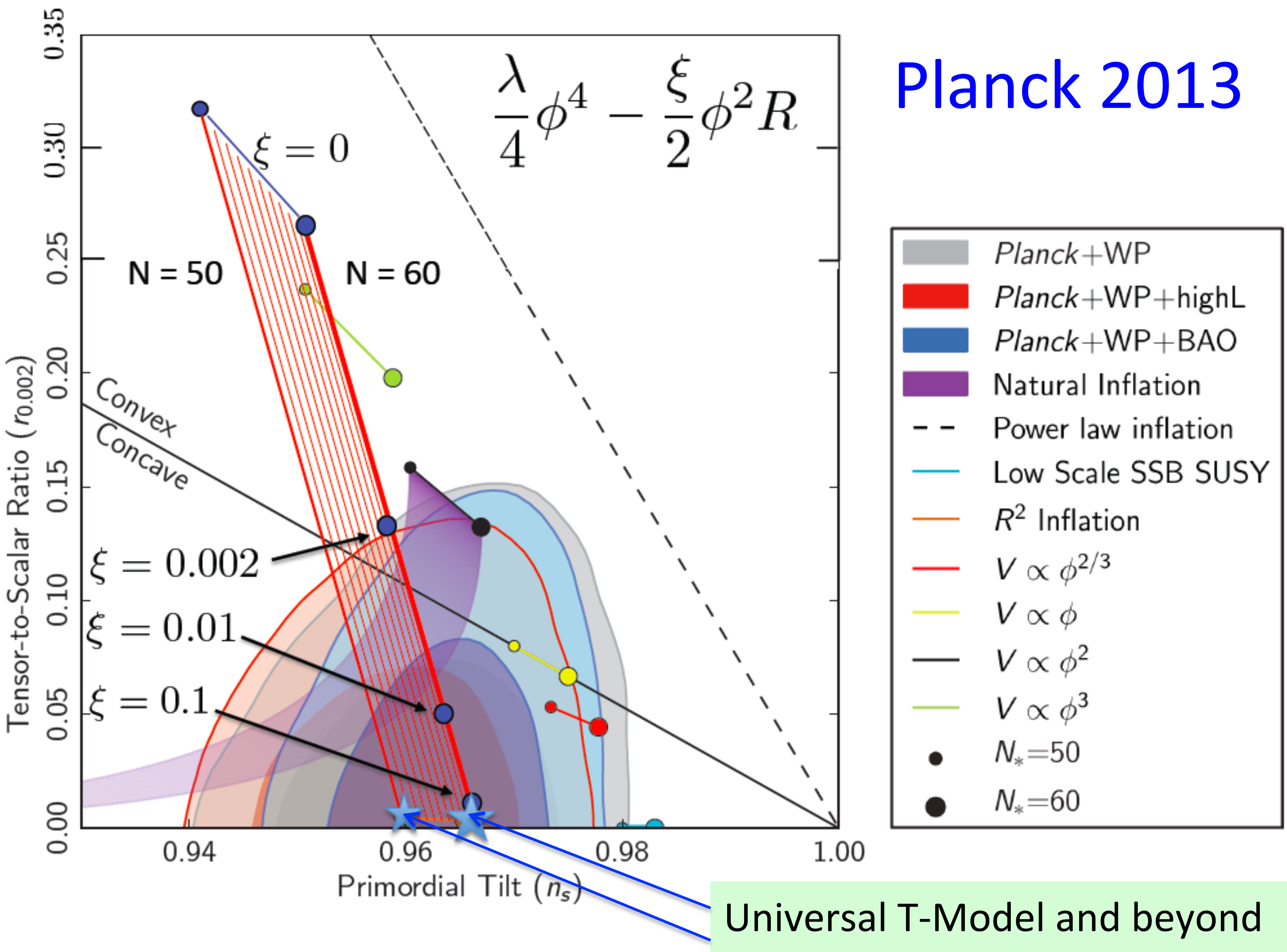
$$V(\phi) \approx V(\sqrt{6})\,(1 - 2\sqrt{6}\,c_1\ e^{-\sqrt{2/3}\,\varphi})$$

Solving equations of motion gives $2\sqrt{6}\,c_1 e^{-\sqrt{2/3}\,\varphi(N)} = \frac{3}{2\,N}$

Therefore it is indeed OK to ignore higher order corrections:

$$V(\phi_N) = V(\sqrt{6})\,(1 - \frac{3}{2\,N} - O(1/N^2))$$

Planck 2013



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N_*=50
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Universal T-Model and beyond

SU(2,2|1) invariant superconformal action

We use 3 superfields: a conformon, an inflaton, and a goldstino

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm sc}^{\rm scalar-grav} = -\tfrac{1}{6}\mathcal{N}(X,\bar{X})R - G_{I\bar{J}}\mathcal{D}^{\mu}X^{I}\,\mathcal{D}_{\mu}\bar{X}^{\bar{J}} - G^{I\bar{J}}\mathcal{W}_{I}\bar{\mathcal{W}}_{\bar{J}}\,,$$

↑ Kahler potential of an embedding manifold

↑ Derivative of the superpotential

$$\mathcal{N}(X,\bar{X}) = -|X^0|^2 + |X^1|^2 + |S|^2 - 3\zeta\,\frac{(S\bar{S})^2}{|X^0|^2 - |X^1|^2}$$

$$\mathcal{W} = S\Big((X^0)^2 - (X^1)^2\Big)f(X^1/X^0)$$

Supergravity implementation

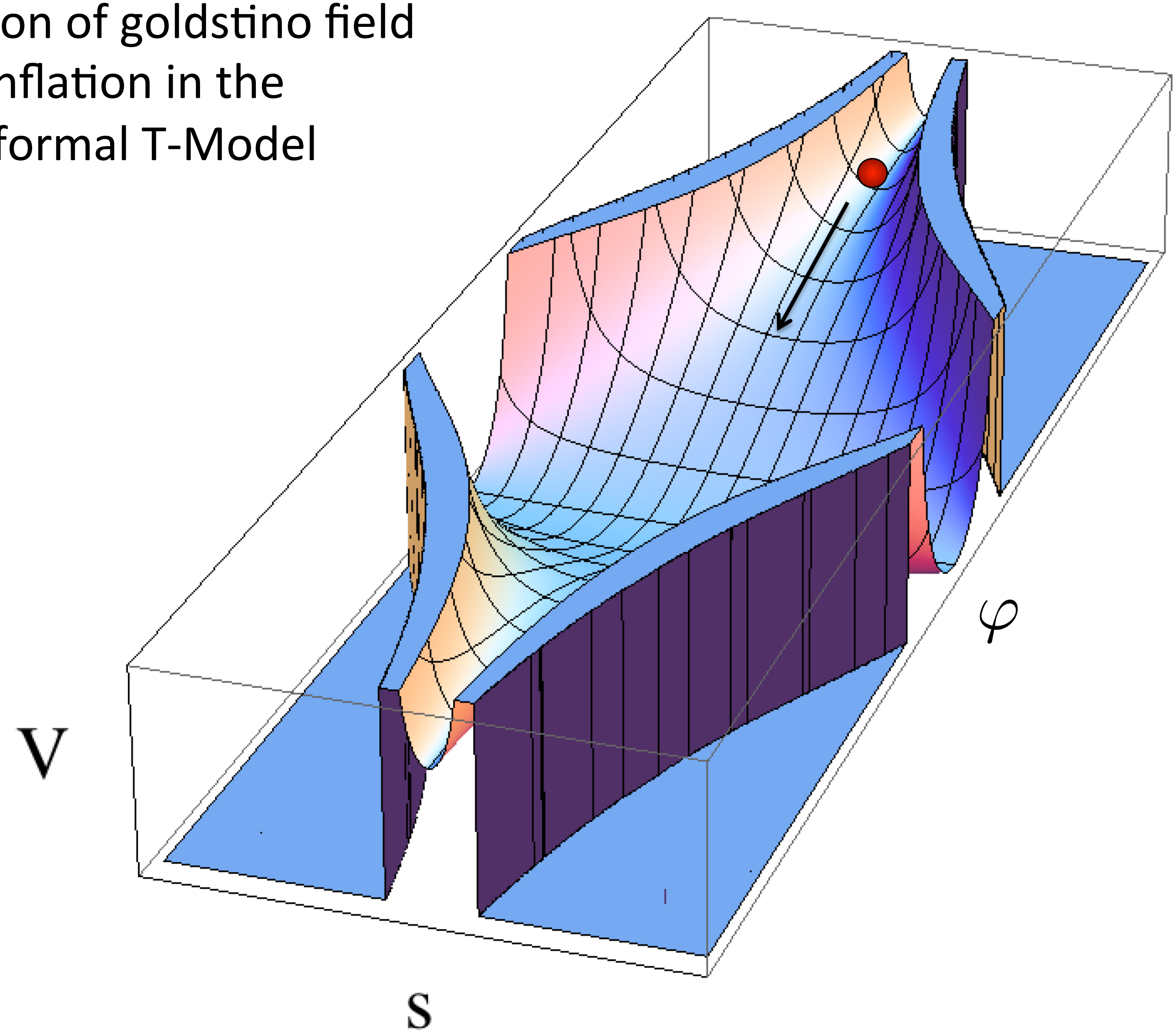
$$K = -3\ln\left[1 - \frac{|\Phi|^2 + |S^2|}{3} + \zeta\,\frac{(S\bar{S})^2}{3 - |\Phi|^2}\right],$$

$$W = S\Big(3 - (\Phi)^2\Big)\, f(\Phi/\sqrt{3})\ .$$

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm sc}^{\rm scalar-grav} = \tfrac{1}{2}R - \frac{1}{2}(\partial^\mu \varphi)^2 - |f(\tanh\varphi)|^2$$

Thus we have the same inflaton potential as in the class of conformal models discussed earlier. Other fields are stabilized at their zero values if ζ > 1/6.

Stabilization of goldstino field S during inflation in the superconformal T-Model



Discussion

Historically, conformally invariant theories were discarded by inflationary theory model builders, just as massless Yang-Mills fields were discarded by phenomenologists until invention of the standard model based on spontaneous symmetry breaking.
It seems now that theories with spontaneously broken conformal or superconformal invariance may play important role in inflationary cosmology.

We developed a new class of chaotic inflation models with spontaneously broken conformal or superconformal invariance. Observational consequences of a broad class of such models are stable with respect to strong deformations of the scalar potential.

Discussion

In this class of models, inflation is possible even in the theories with very steep potentials because of their exponential flattening at the boundary of the moduli space. This effect can be described as inflation of moduli space, which exponentially stretches and flattens the inflaton potential. In this sense, slow roll inflation **in** the landscape is facilitated by inflation **of** the landscape.

We found a broad class of models of that type belonging to a certain universality class, with the same observational predictions,

$$1 - n_s = 2/N\,, \qquad r = 12/N^2$$

which are favored by the Planck data. Experience with black hole attractors suggests that this is just a first step in finding other, more general types of cosmological attractors.