

Refined BPS invariants of del Pezzo and half K3 manifolds

Based on: M. X. Huang, A. Klemm, M. P.: arXiv:1306.????[hep-th]

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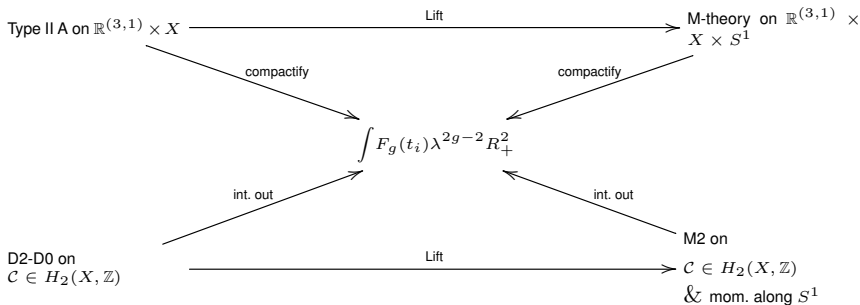


Computation of refined BPS numbers

We have computed the index

$$I = \text{Tr}_{BPS}(-1)^{2(J_L+J_R)} e^{-2\epsilon_L J_L} e^{-2\epsilon_R J_R} e^{-2\epsilon_R J_R} e^{\beta H}$$

that counts the refined BPS numbers N_{j_L, j_R}^β for local CY - geometries (X) over del Pezzo and half K3 manifolds.



$$\lambda = \langle \epsilon_1 dx_1 \wedge dx_2 + \epsilon_2 dx_3 \wedge dx_4 \rangle + \delta \lambda$$

Appetizer on some explicit results

$2j_L \setminus 2j_R$	0	1
0	248	
1		1

$d = 1$

$2j_L \setminus 2j_R$	0	1	2	3
0		3876		
1			248	
2				1

$d = 2$

$2j_L \setminus 2j_R$	0	1	2	3	4	5	6
0	30628		151374		248		
1		4124		34504		1	
2	1		248		4124		
3				1		248	
4							1

$d = 3$

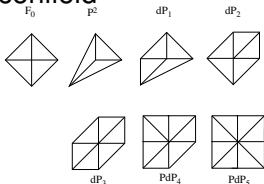
The BPS invariants N_{j_L, j_R}^d for $d = 1, 2, 3$ for the local E_8 del Pezzo surface.

Calculation is possible by use of anomalies

Generalized holomorphic anomaly

Most important ingredients:

- Weierstrass form of B-model geometry
- Heavily use that B-model geometry has just one complex modulus
- Gap condition at the conifold



Generalized modular anomaly

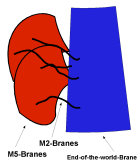
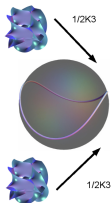
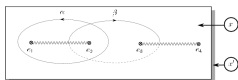
Most important ingredients:

- Homology lattice of del Pezzo surface contains weight lattice of exceptional Lie groups
- Express free energy in terms of modular forms w.r.t. these lattices



Krefl, Walcher '10; Huang, Klemm '10

Results are linked to various applications



- Seiberg Witten theory with matter
- Geometries appear for stable degeneration limit of F-theory
- Counting small sized instantons in heterotic string theory

Witten '95

- Connection to E-, (p,q)- and eventually M-Strings

Vafa et al.; Gaberdiel, Zwiebach '97

- Cohomological invariants of stable pairs

Choi, Katz, Klemm '12

We have presented a framework to compute refined BPS numbers that

- works everywhere in the moduli space
- works for toric del Pezzo surfaces
- works for non-toric del Pezzo surfaces

Outlook:

- Elaborate on the connection to E- and M-Strings.
- Improve the development of the corresponding mathematical theory