Imperial College London

Recent Progress on Brane Tilings

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Double Handled Brane Tilings Cremonesi, Hanany, Seong
1305.3607

New Directions in Bipartite Field Theories Franco, Galloni, Seong
1211.5139 JHEP 1306 (2013) 032

Brane Tilings and Specular Duality Hanany, Seong
1206.2386 JHEP 1208 (2012) 107

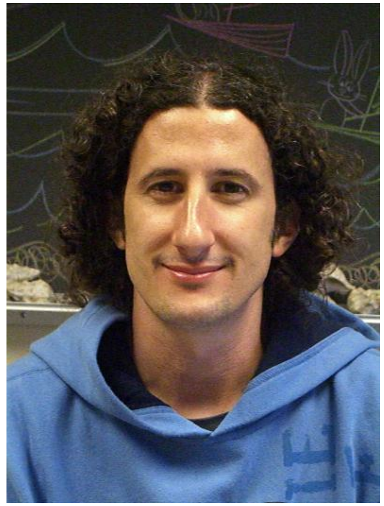
Brane Tilings and Reflexive Polygons Hanany, Seong
1201.2614 Fortschritte der Physik 60 (2012) 695-803

Calabi-Yau Orbifolds and Torus Coverings Hanany, Jejjala, Ramgoolam, Seong
1105.3471 JHEP 1109 (2011) 116

Symmetries of Abelian Orbifolds Hanany, Seong
1009.3017 JHEP 1101 (2011) 027

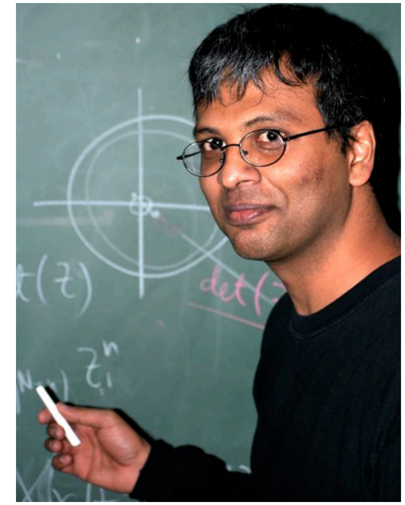
Counting Orbifolds Davey, Hanany, Seong
1002.3609 JHEP 1006 (2010) 010

Amihay Hanany



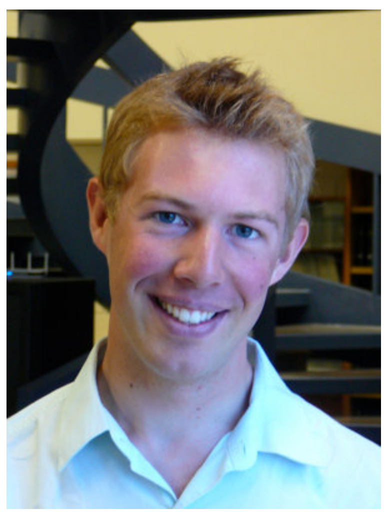
Sebastian Franco

Sanjaye Ramgoolam



Vishnu Jejjala

Stefano Cremonesi



John Davey

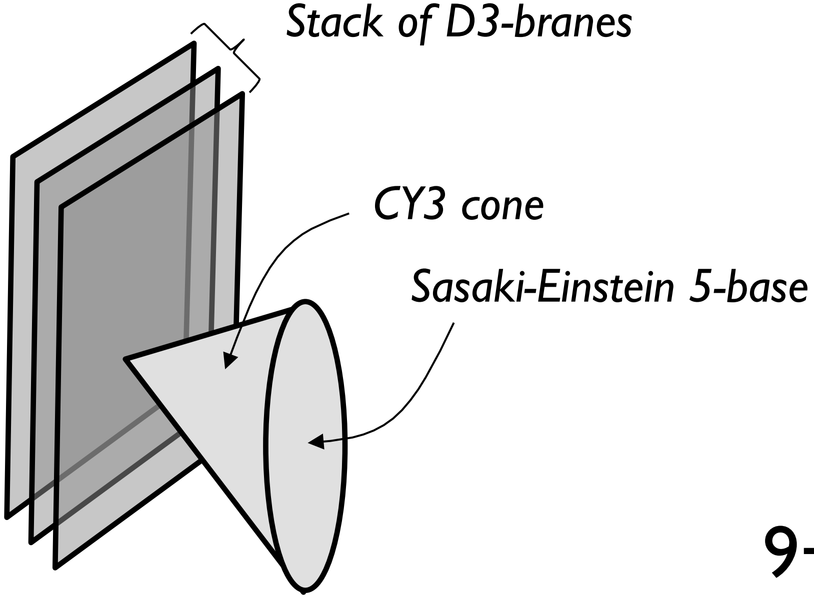
Daniele Galloni

The Theories

4d N=1 Supersymmetric Gauge Theory

product gauge groups $U(N)^G$ Quiver Superpotential

superconformal



worldvolume theory of a stack of *N* D3 branes probing singular CY

9+1d Type IIB on $AdS_5 \times SE^5$

The Theories

4d N=1 Supersymmetric Gauge Theory

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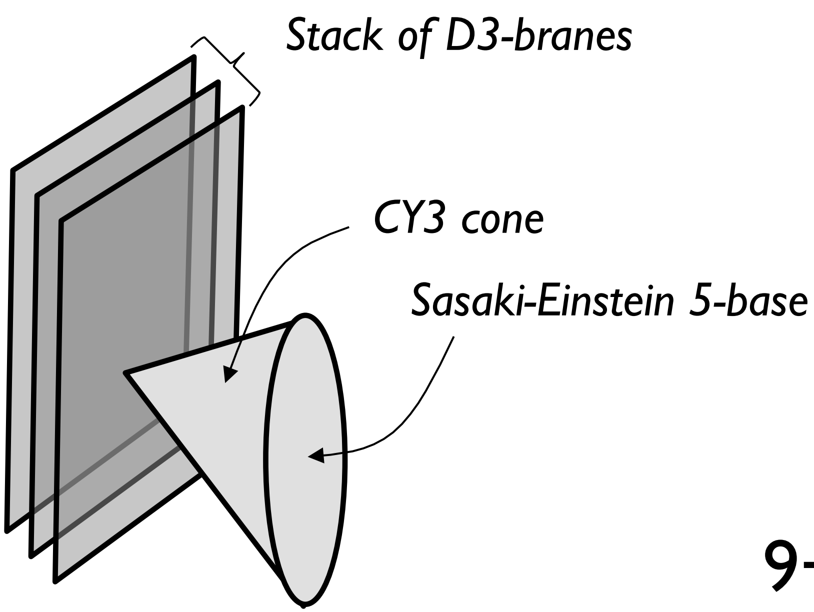


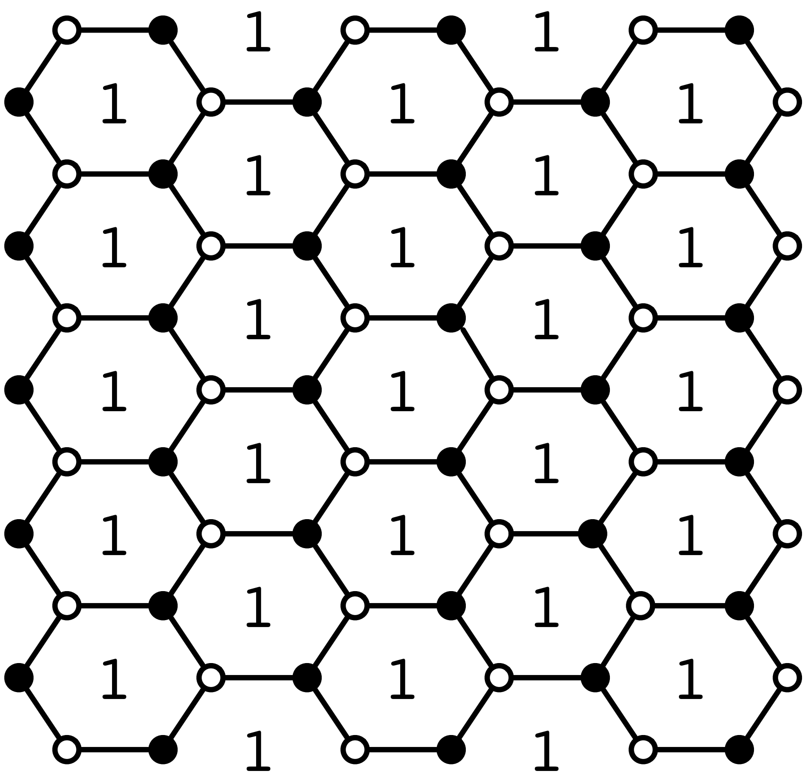
worldvolume theory of a stack of *N* D3 branes probing singular CY

9+1d Type IIB on $AdS_5 \times SE^5$

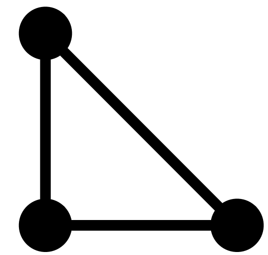
#gauge groups - #chiral matter fields + #superpotential terms = 0

Brane Tiling $\mathbb{C}^3$

bipartite periodic graph on the 2-torus



Toric Diagram



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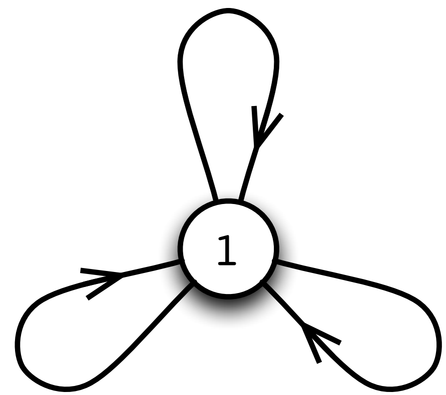


Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

4d SYM is represented by a hexagonal tiling

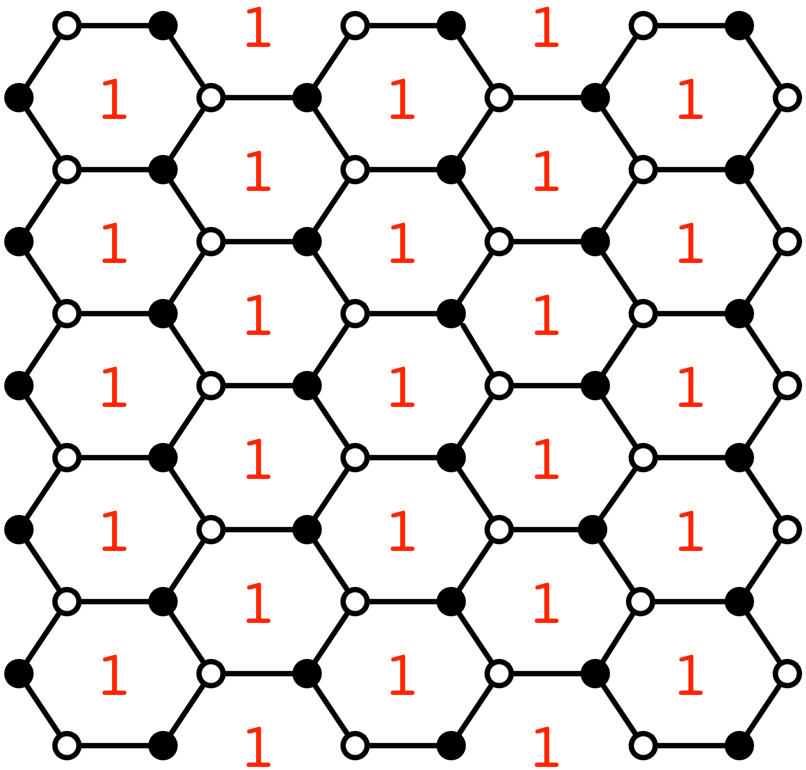
Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

Hanany, Kennaway 2005

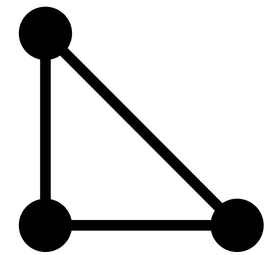
1

Brane Tiling $\mathbb{C}^3$

faces represent U(N) gauge groups



Toric Diagram



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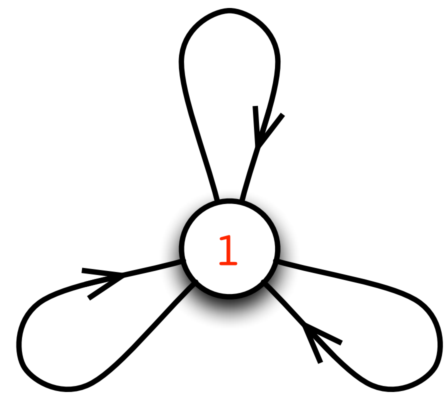


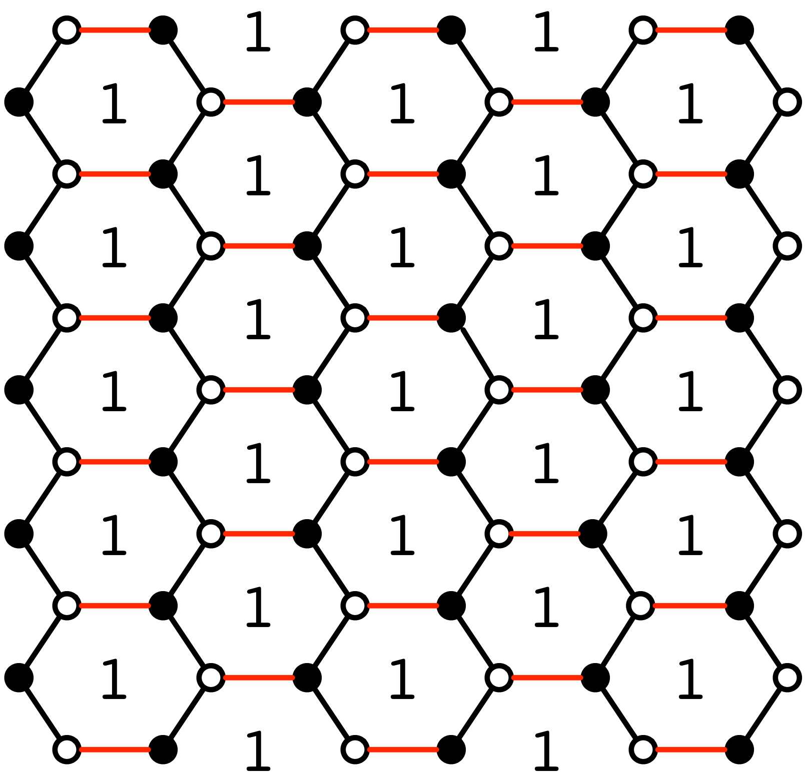
Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

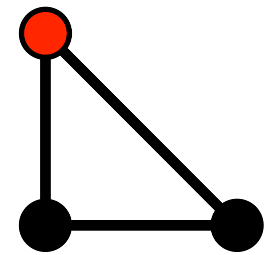
Hanany, Kennaway 2005

Brane Tiling $\mathbb{C}^3$

edges correspond to bifundamental matter fields



Toric Diagram



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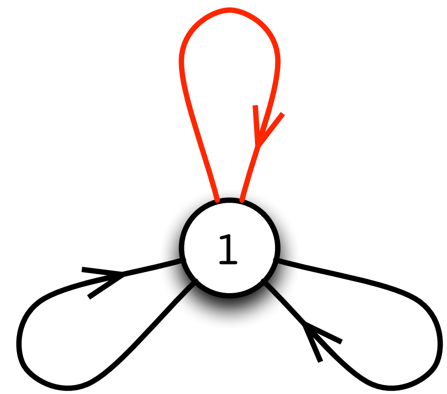

Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

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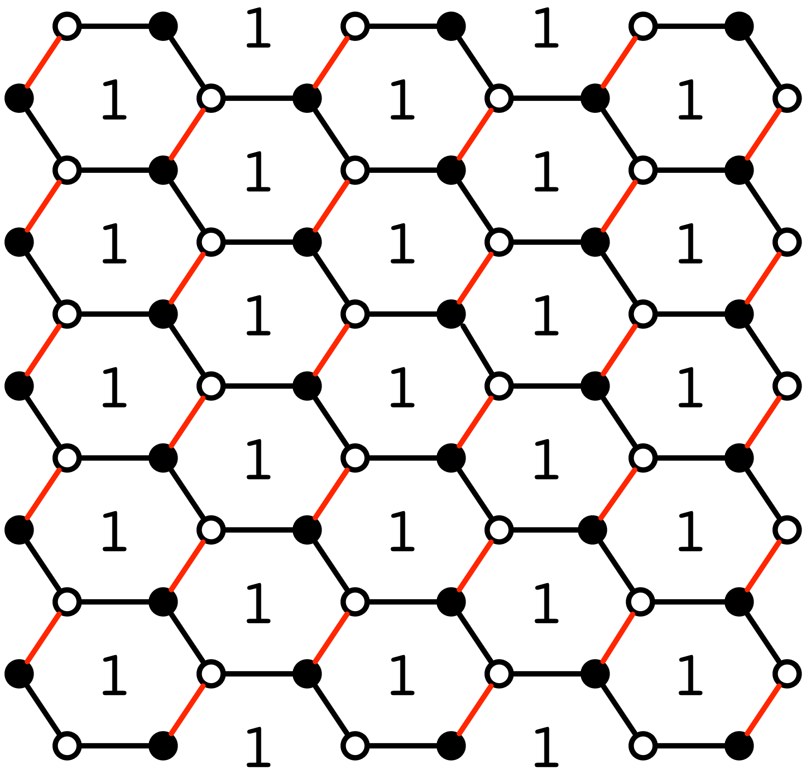
Hanany, Kennaway 2005

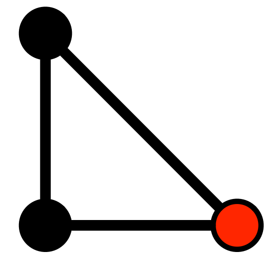
1

Brane Tiling $\mathbb{C}^3$

edges correspond to bifundamental matter fields



Toric Diagram



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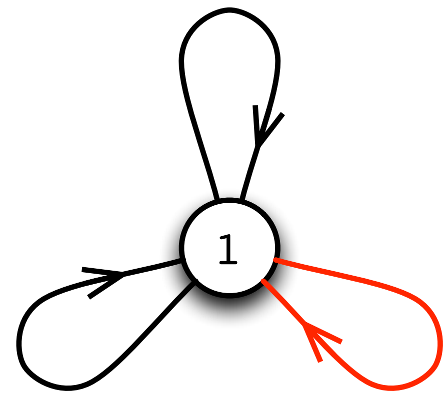


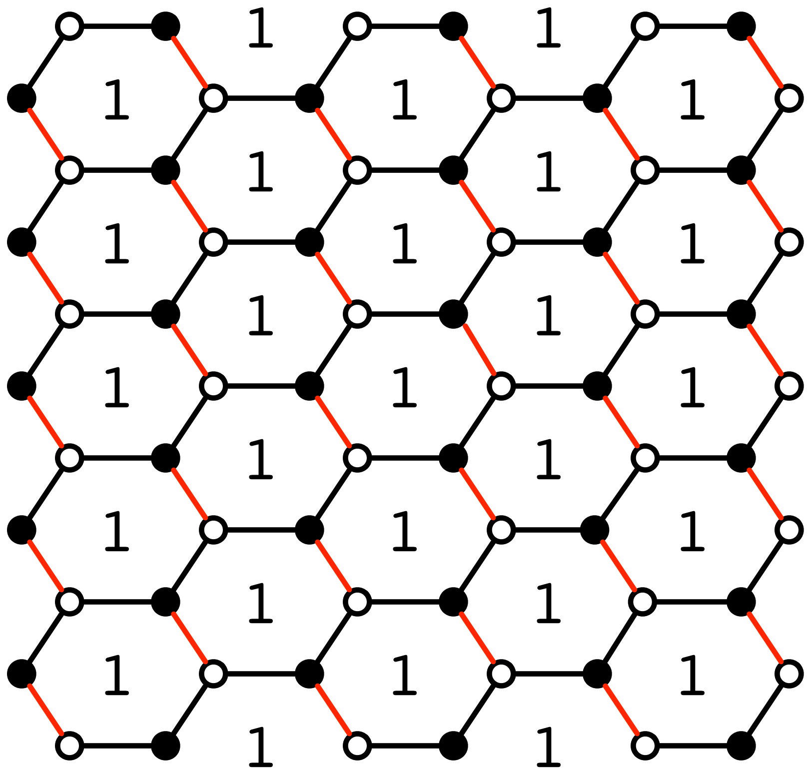
Superpotential $W = \phi_1 \phi_2 \phi_3 - \phi_1 \phi_3 \phi_2$

Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

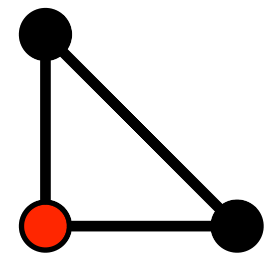
Hanany, Kennaway 2005

Brane Tiling $\mathbb{C}^3$

edges correspond to bifundamental matter fields



Toric Diagram



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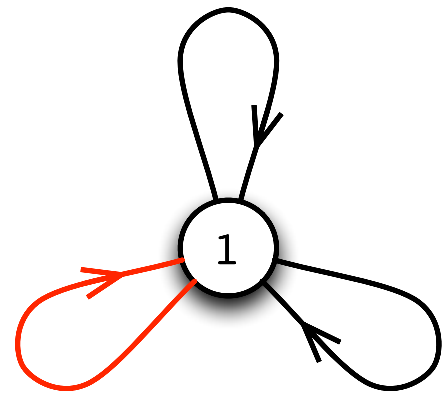


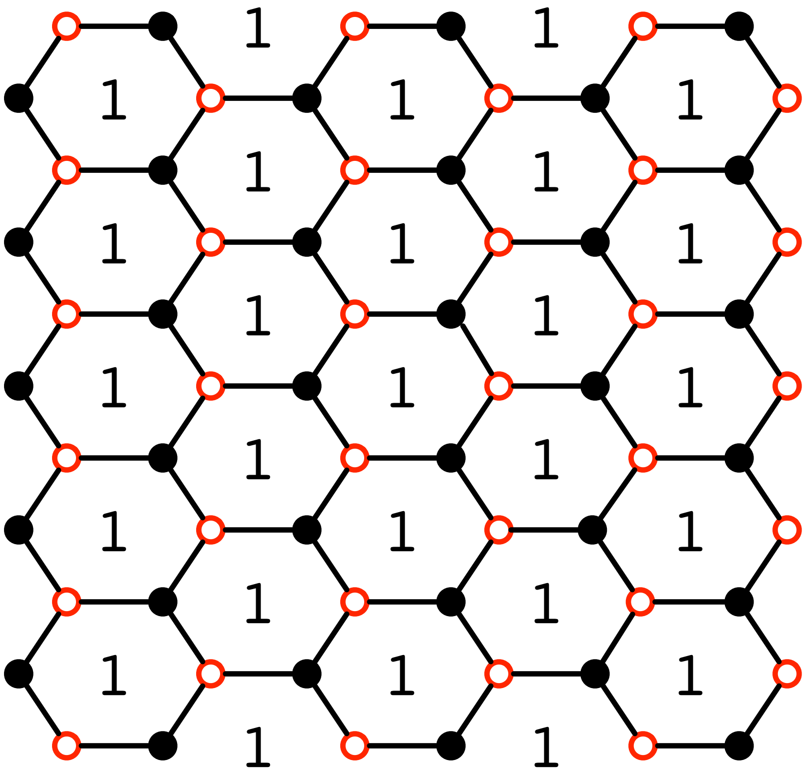
Superpotential $W = \phi_1 \phi_2 \phi_3 - \phi_1 \phi_3 \phi_2$

Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

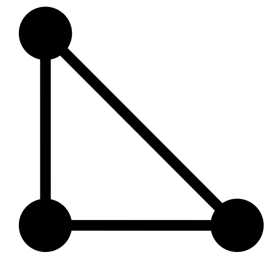
Hanany, Kennaway 2005

Brane Tiling $\mathbb{C}^3$

white nodes correspond to +ve superpotential terms



Toric Diagram



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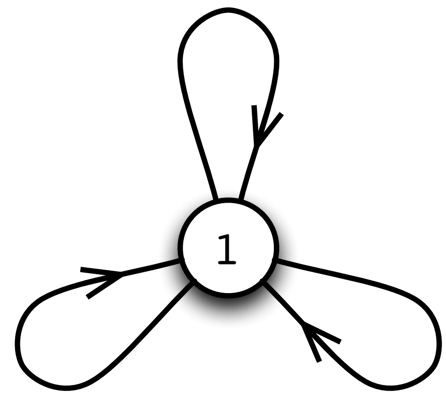


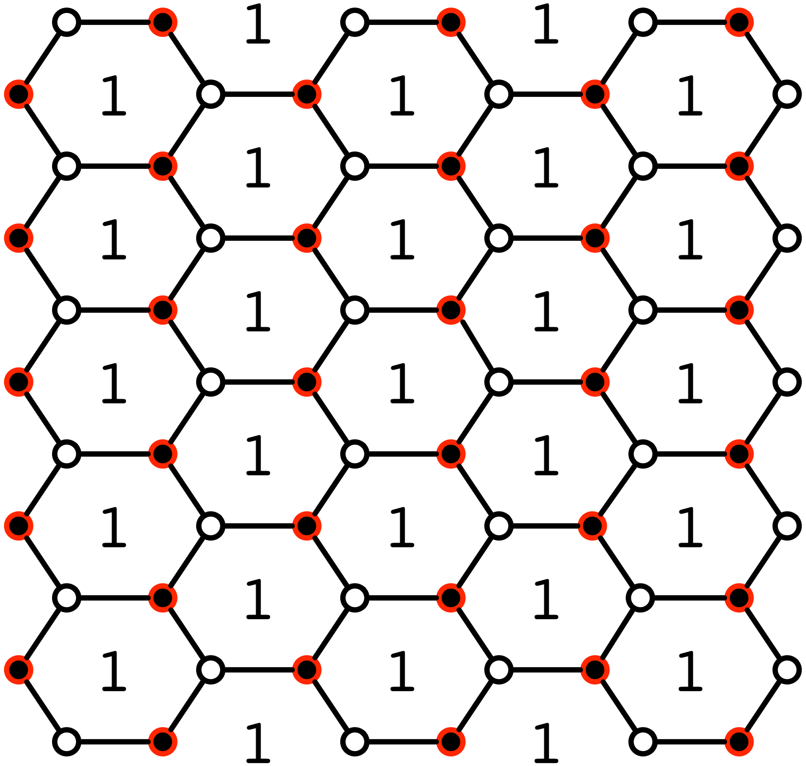
Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

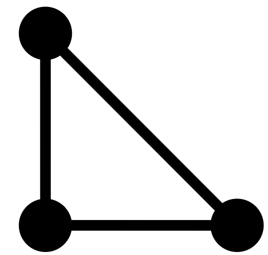
Hanany, Kennaway 2005

Brane Tiling $\mathbb{C}^3$

black nodes correspond to -ve superpotential terms



Toric Diagram



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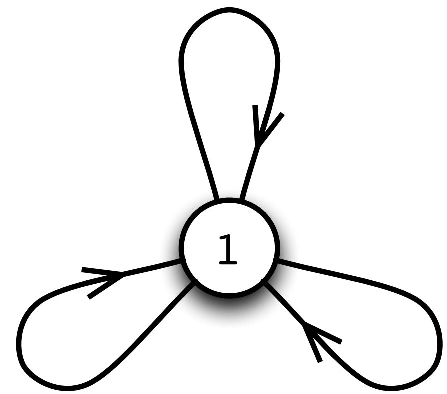
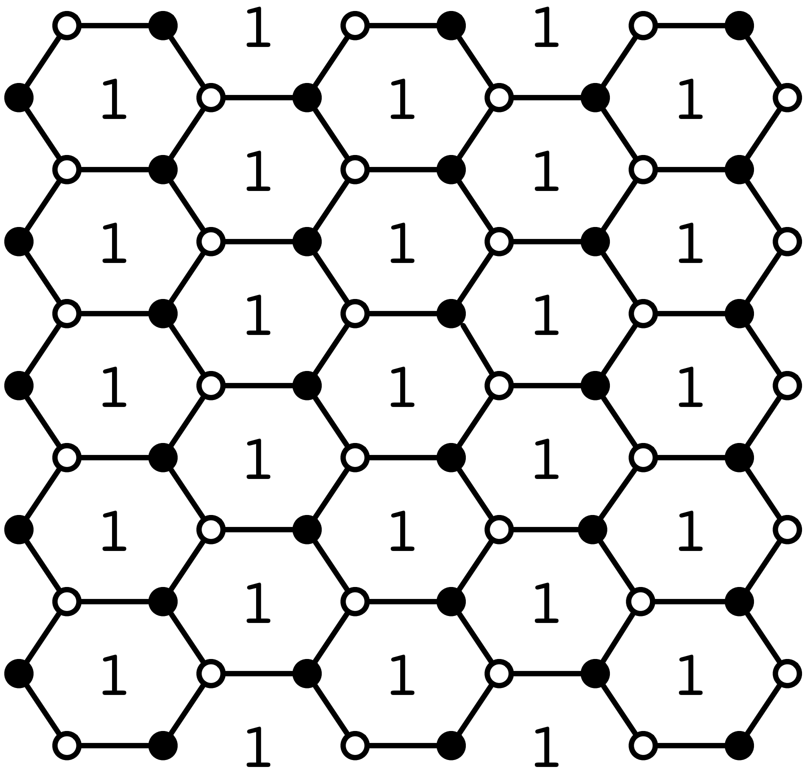


Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

Hanany, Kennaway 2005

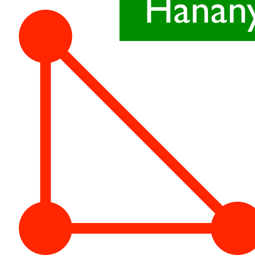
Brane Tiling $\mathbb{C}^3$



Benvenuti, Feng, Hanany, He 2007

Hanany, Kennaway 2005

Toric Diagram



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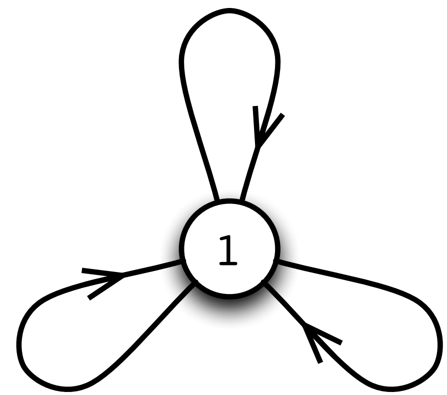


Superpotential $W = \phi_1\phi_2\phi_3 - \phi_1\phi_3\phi_2$

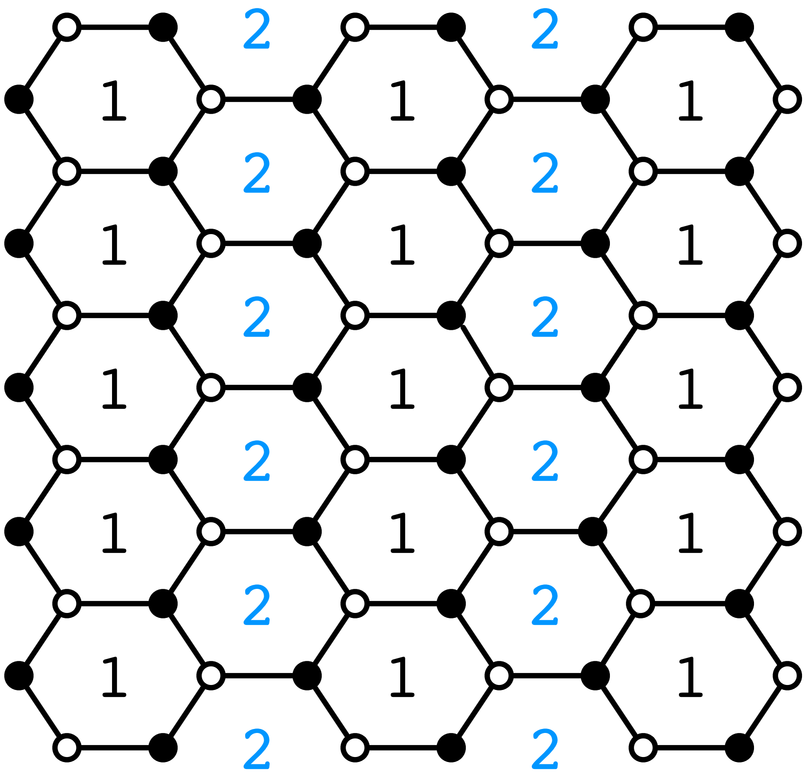
Hilbert Series (mesonic) $g(x_i, t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n, 0] t^n$

$SU(3)_x$ → $[n, 0]$; $U(1)_R$ → t^n

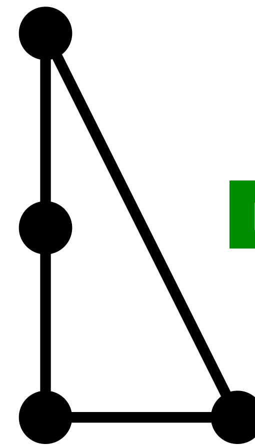
the mesonic Hilbert series is a partition function of mesonic GIOs

Orbifolding $\mathbb{C}^3/\mathbb{Z}_2$

by relabelling faces, one obtains Abelian orbifolds

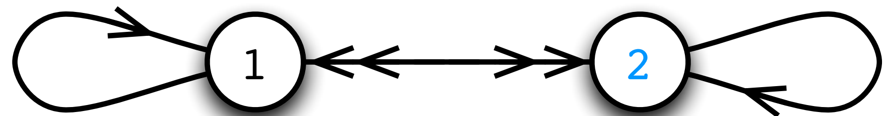


Toric Diagram



Davey, Hanany, Seong 2010

Hanany, Orlando, Reffert 2010

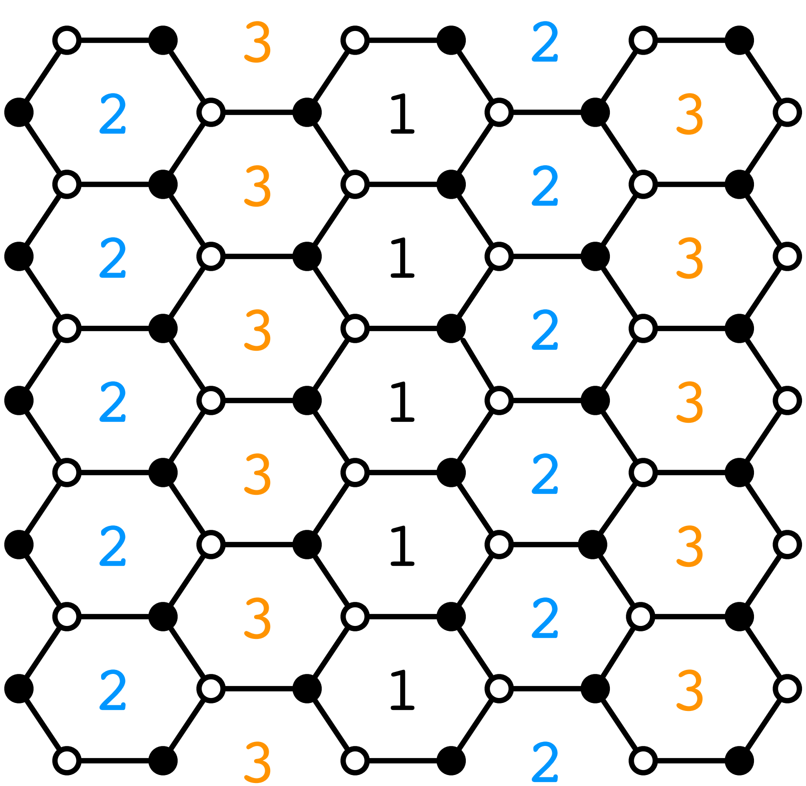


$$W = X_{21}\phi_1 X_{12} + Y_{12}\phi_2 Y_{21} - X_{12}\phi_2 X_{21} - Y_{21}\phi_1 Y_{12}$$

$$g(x_i, t; \mathcal{M}^{mes}) = \frac{1}{1-t}\sum_{n=0}^{\infty}[2n, 0]t^{2n}$$

Orbifolding $\mathbb{C}^3/\mathbb{Z}_3$

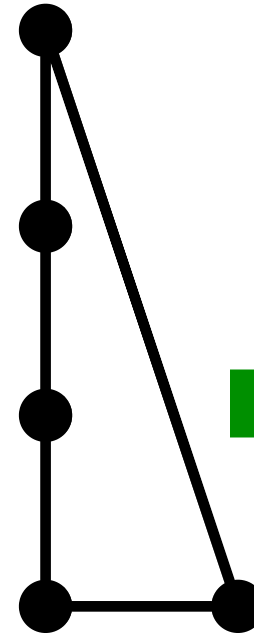
by relabelling faces, one obtains Abelian orbifolds

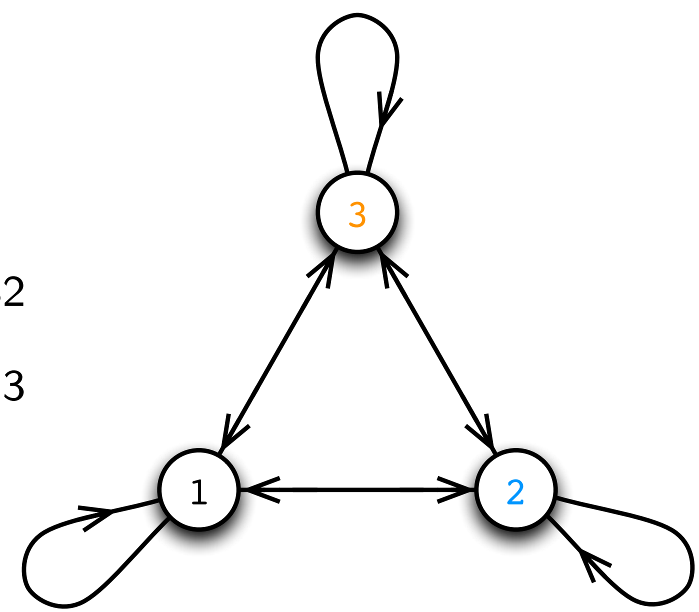


Davey, Hanany, Seong 2010

Hanany, Orlando, Reffert 2010

Toric Diagram





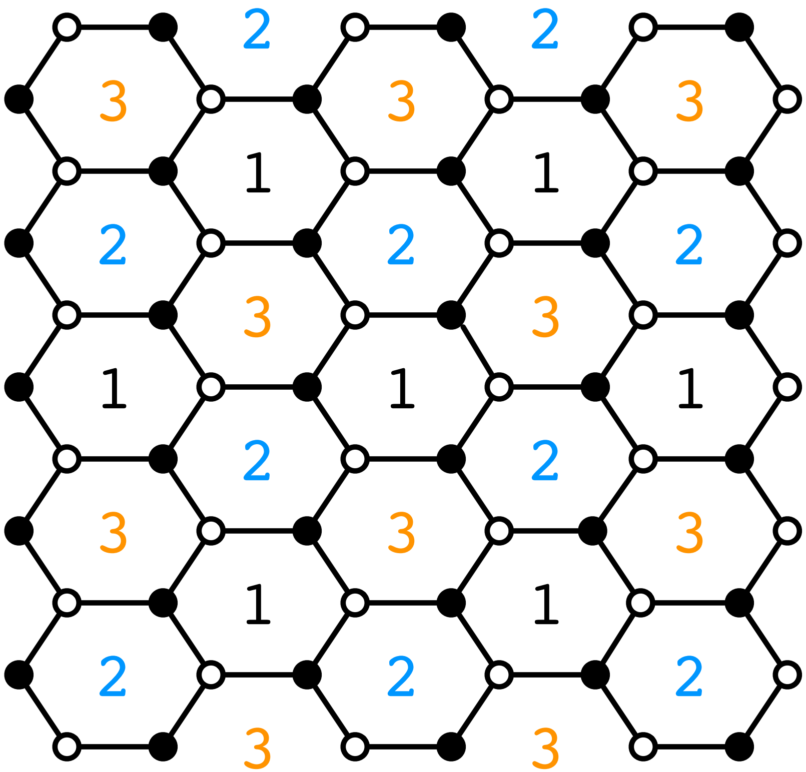
$$W = X_{31}\phi_1 X_{13} + X_{12}\phi_2 X_{21} + X_{23}\phi_3 X_{32} - X_{13}\phi_3 X_{31} - X_{21}\phi_1 X_{12} - X_{32}\phi_2 X_{23}$$

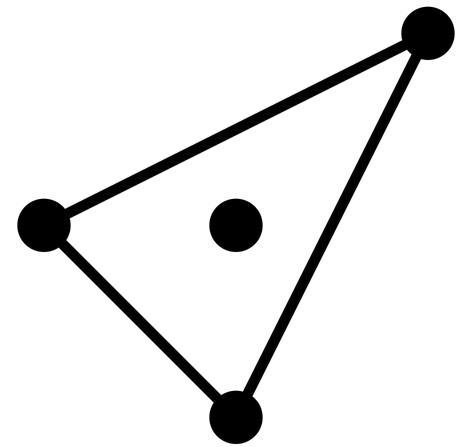
$$g(t;\mathcal{M}^{mes}) = \frac{1+t^2+t^4}{(1-t)(1-t^3)^2}$$

Orbifolding

$\mathbb{C}^3/\mathbb{Z}_3$

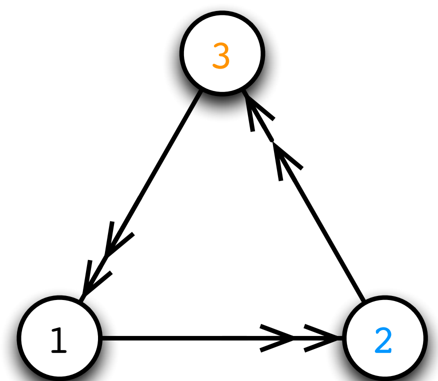
by relabelling faces, one obtains Abelian orbifolds



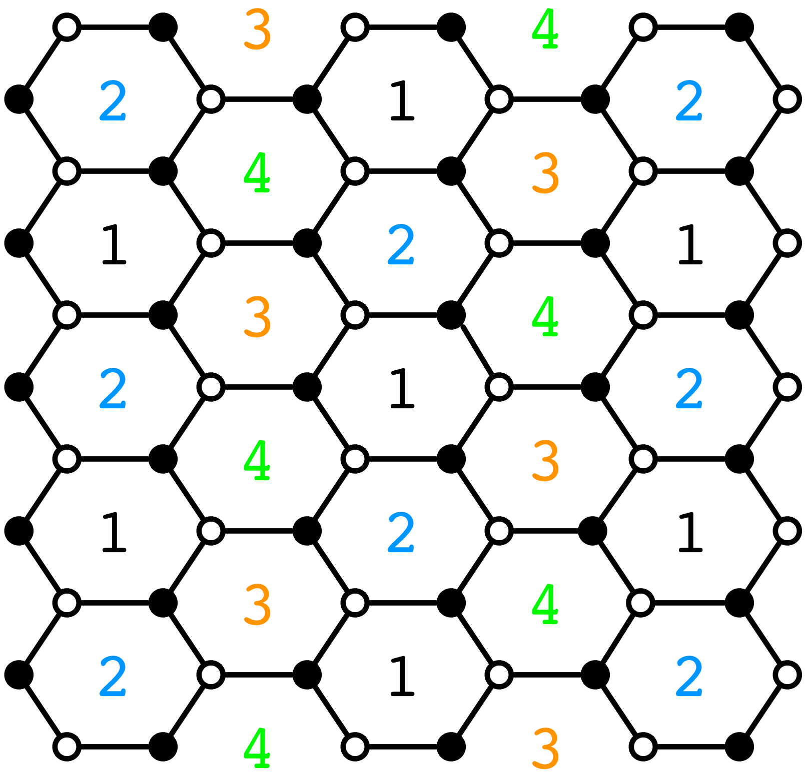


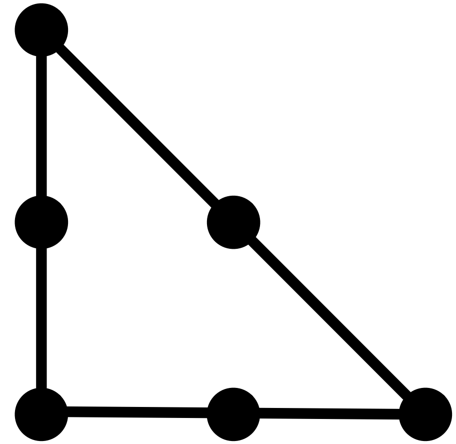
$$W = X_{13}X_{32}X_{21} + Y_{13}X_{32}Y_{21} + Z_{13}Z_{32}Z_{21} - Y_{21}Z_{13}X_{32} - X_{21}Y_{13}Z_{32} - Z_{21}X_{13}Y_{32}$$

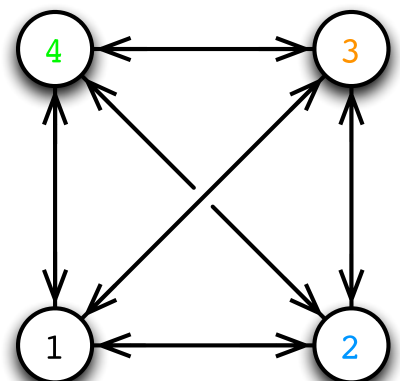
$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[3n,0]t^{3n}$$



Orbifolding $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$



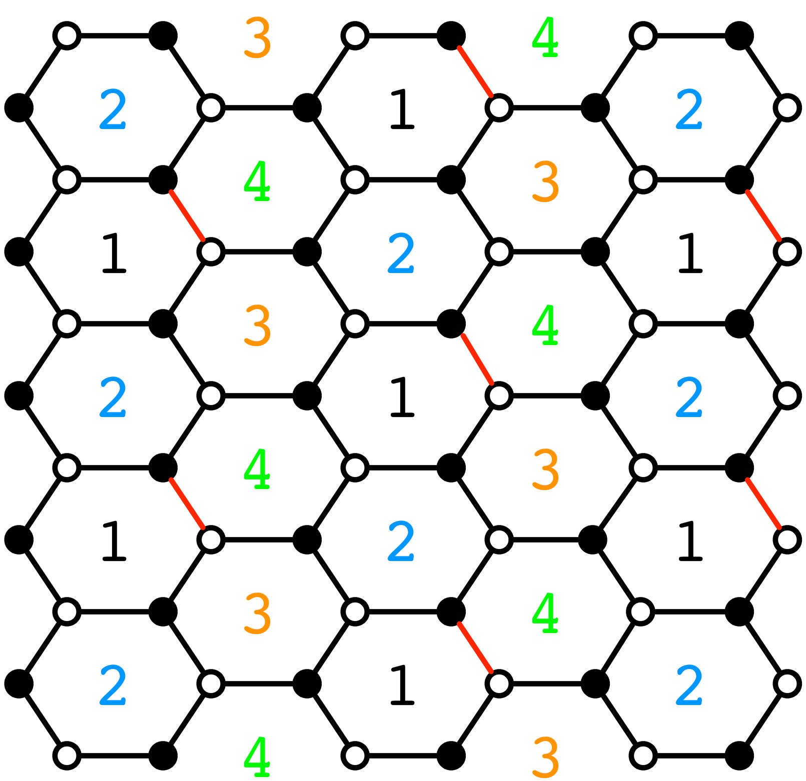


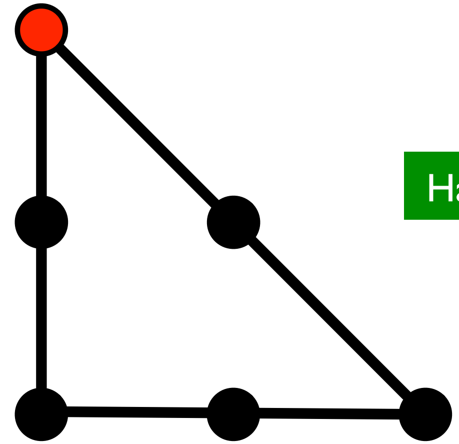


$$W = X_{14}X_{43}X_{31} + X_{13}X_{32}X_{21} + X_{42}X_{23}X_{34} + X_{24}X_{41}X_{12}$$
$$-X_{13}X_{34}X_{41} - X_{31}X_{12}X_{23} - X_{24}X_{43}X_{32} - X_{42}X_{21}X_{14}$$

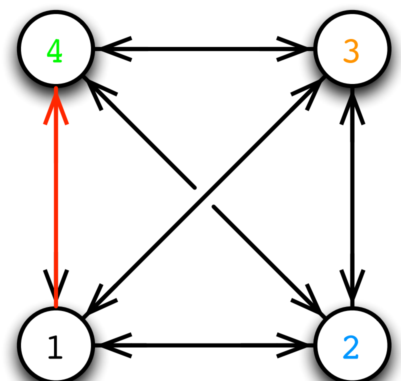
$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [3n, 0] t^{3n}$$

Higgsing $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$





Feng, Franco, Hanany, He 2002

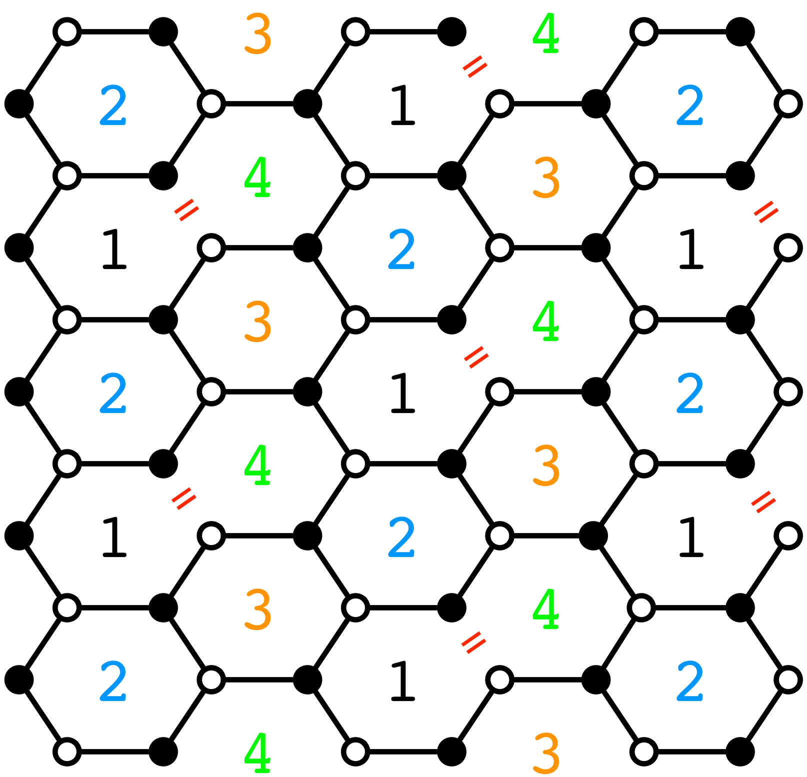


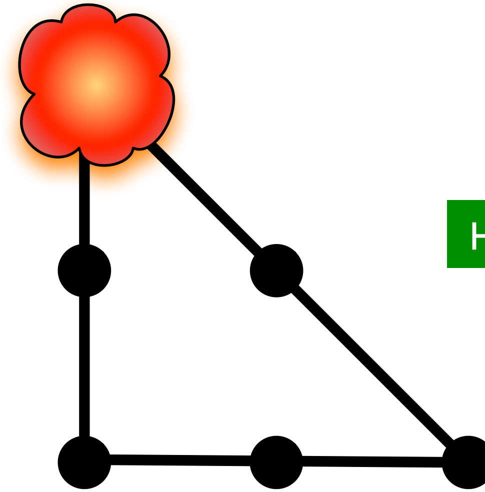
$$W = X_{14}X_{43}X_{31} + X_{13}X_{32}X_{21} + X_{42}X_{23}X_{34} + X_{24}X_{41}X_{12}$$
$$- X_{13}X_{34}X_{41} - X_{31}X_{12}X_{23} - X_{24}X_{43}X_{32} - X_{42}X_{21}X_{14}$$

$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [3n, 0] t^{3n}$$

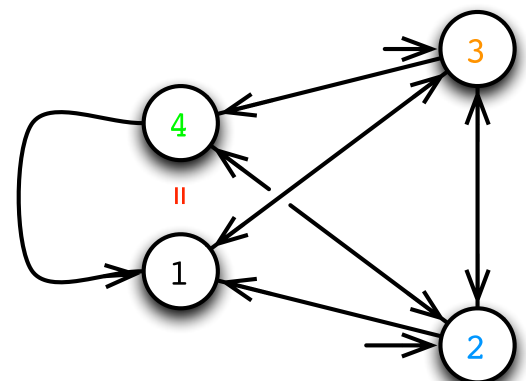
removing an edge corresponds to giving a VEV to a quiver field

Higgsing





Feng, Franco, Hanany, He 2002

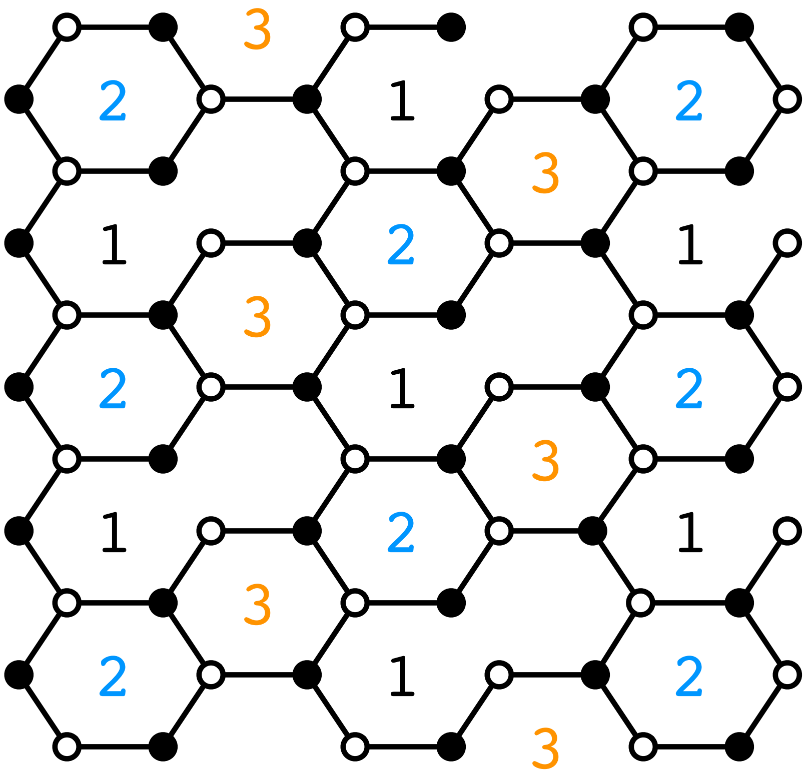


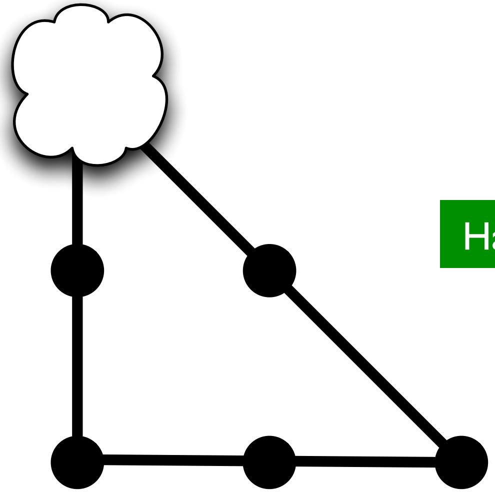
$$W = {\color{red}X_{14}} X_{43} X_{31} + X_{13} X_{32} X_{21} + X_{42} X_{23} X_{34} + X_{24} X_{41} X_{12}$$
$$- X_{13} X_{34} X_{41} - X_{31} X_{12} X_{23} - X_{24} X_{43} X_{32} - X_{42} X_{21} {\color{red}X_{14}}$$

$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [3n, 0] t^{3n}$$

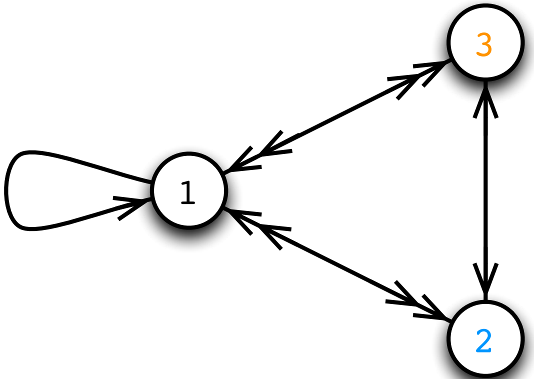
the adjacent gauge groups combine to a single gauge group

Higgsing





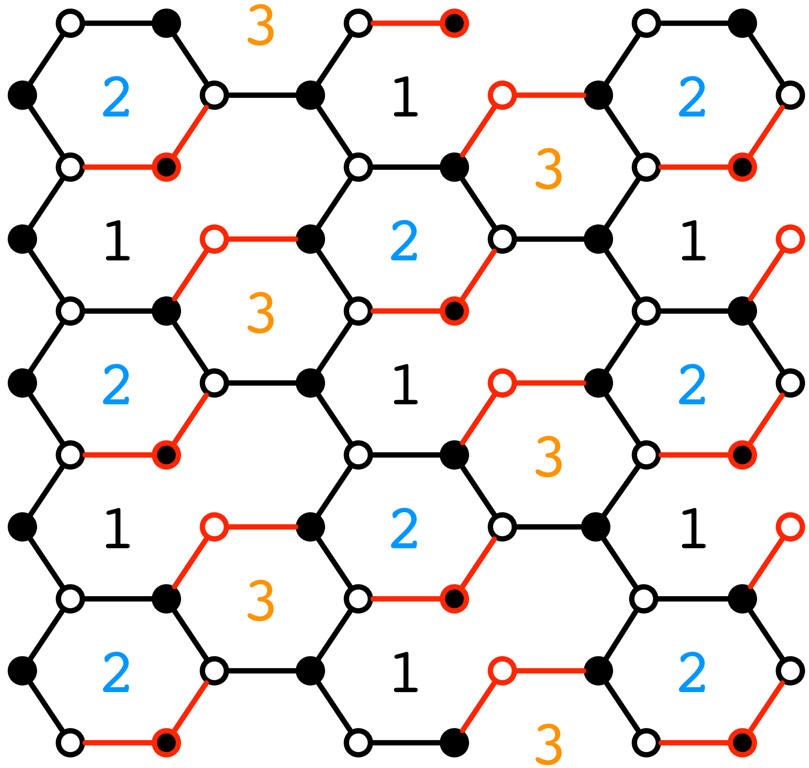
Feng, Franco, Hanany, He 2002

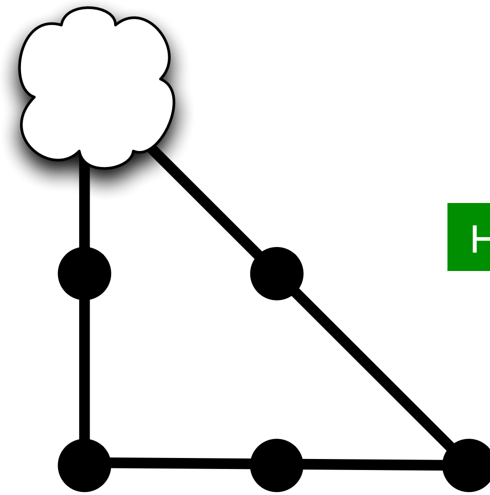


$$W = Y_{13}X_{31} + X_{13}X_{32}X_{21} + Y_{12}X_{23}Y_{31} + Y_{21}\phi X_{12} - X_{13}Y_{31}\phi - X_{31}X_{12}X_{23} - Y_{21}Y_{13}X_{32} - Y_{12}X_{21}$$

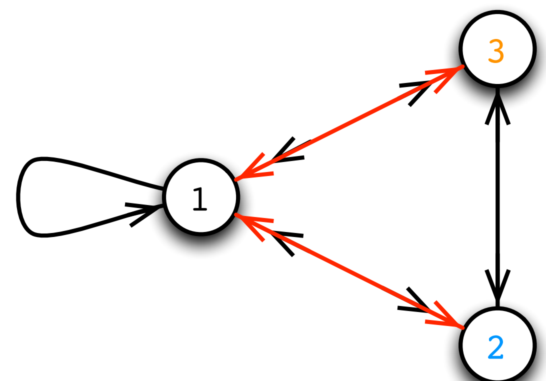
$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[3n,0]t^{3n}$$

Higgsing





Feng, Franco, Hanany, He 2002

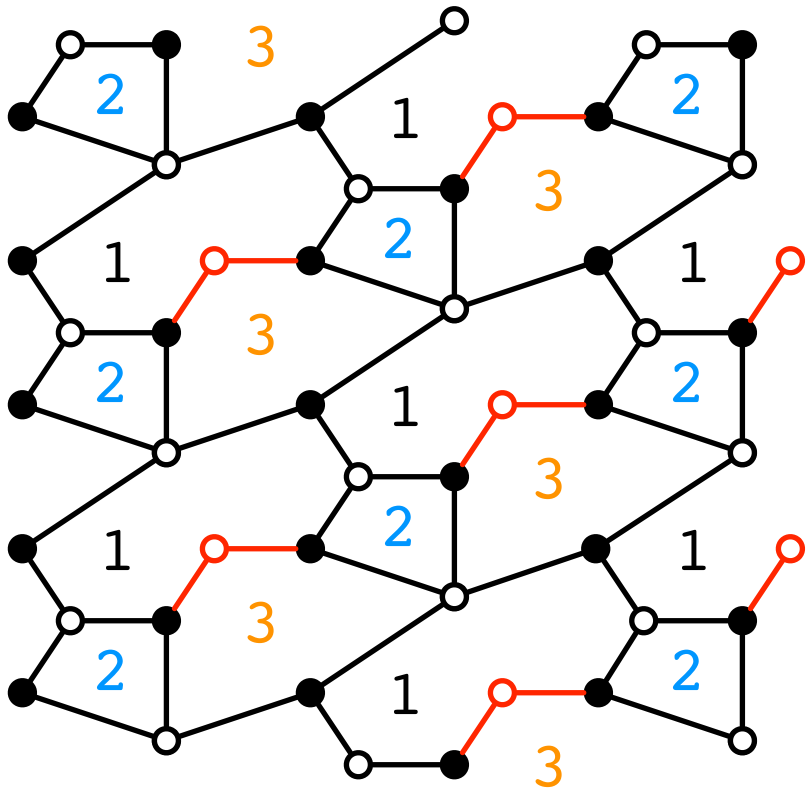


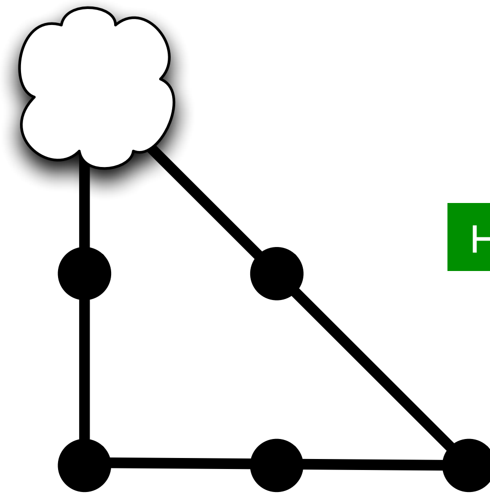
$$W = Y_{13}X_{31} + X_{13}X_{32}X_{21} + Y_{12}X_{23}Y_{31} + Y_{21}\phi X_{12}$$
$$- X_{13}Y_{31}\phi - X_{31}X_{12}X_{23} - Y_{21}Y_{13}X_{32} - Y_{12}X_{21}$$

$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [3n, 0] t^{3n}$$

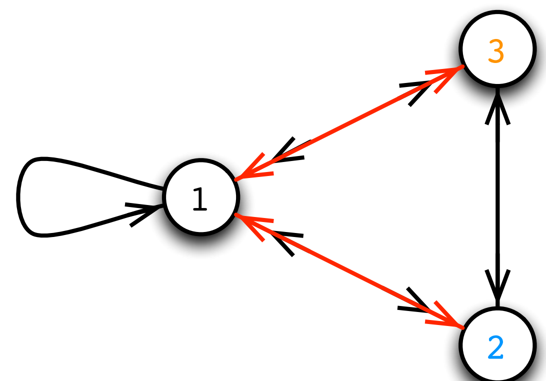
2-valent nodes correspond to quadratic mass terms which are integrated out

Higgsing





Feng, Franco, Hanany, He 2002



$$W = {\color{red}Y_{13}X_{31}} + X_{13}X_{32}X_{23}Y_{31} + Y_{21}\phi X_{12}$$
$$- X_{13}Y_{31}\phi - X_{31}X_{12}X_{23} - Y_{21}Y_{13}X_{32}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[3n,0]t^{3n}$$

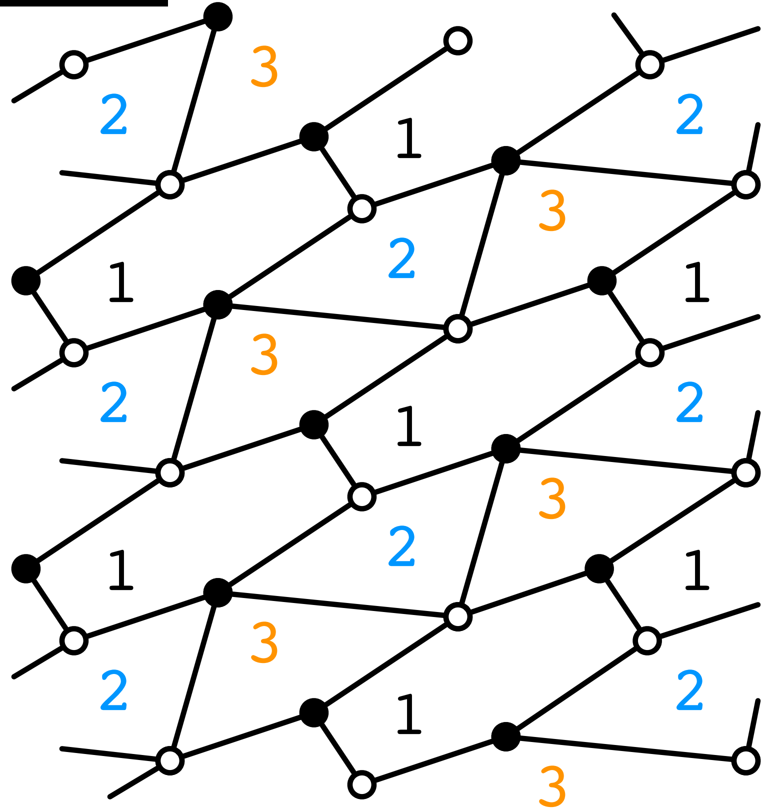
2-valent nodes correspond to quadratic mass terms which are integrated out

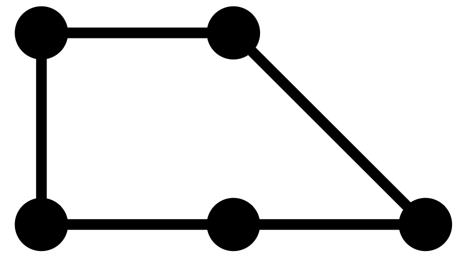
Higgsing

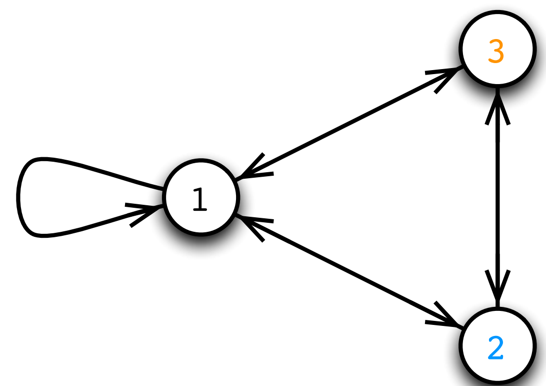
SPP

suspended pinch point theory

Morrison, Plesser 1998







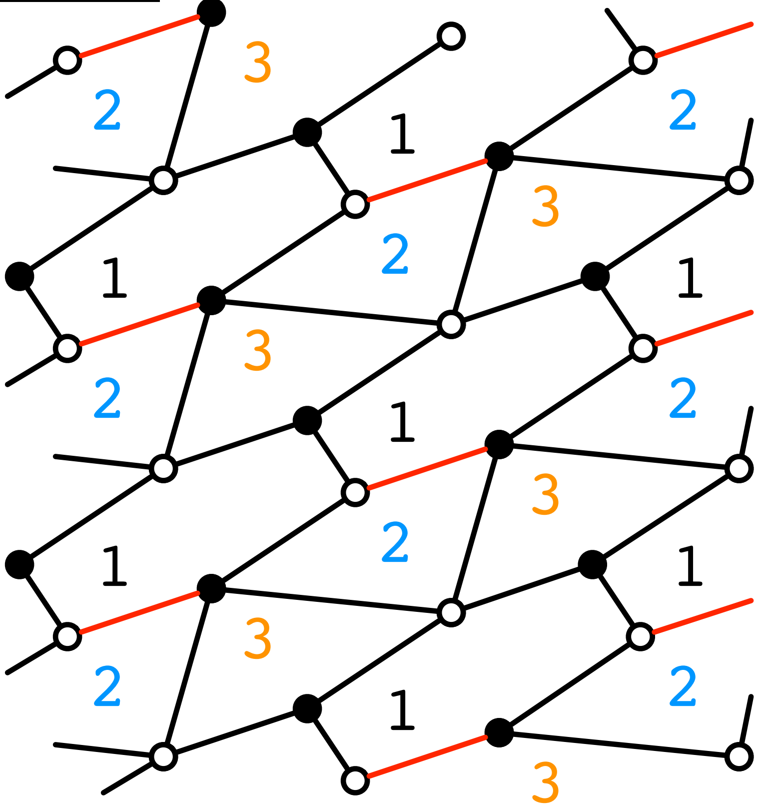
$$W = X_{13}X_{32}X_{23}Y_{31} + Y_{21}\phi X_{12} - X_{13}Y_{31}\phi - X_{12}X_{23}X_{32}Y_{21}$$

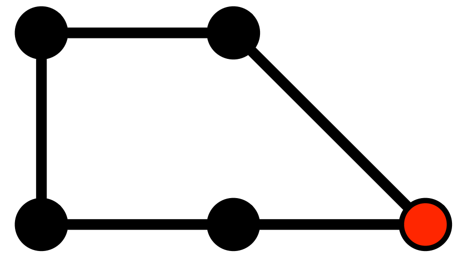
$$g(t;\mathcal{M}^{mes}) = \frac{1-t^6}{(1-t^2)^2(1-t^3)^2}$$

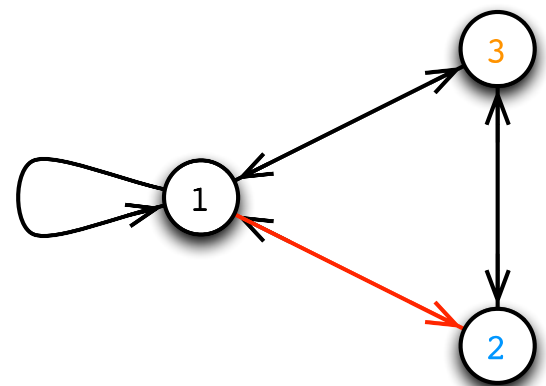
Higgsing

SPP

Morrison, Plesser 1998





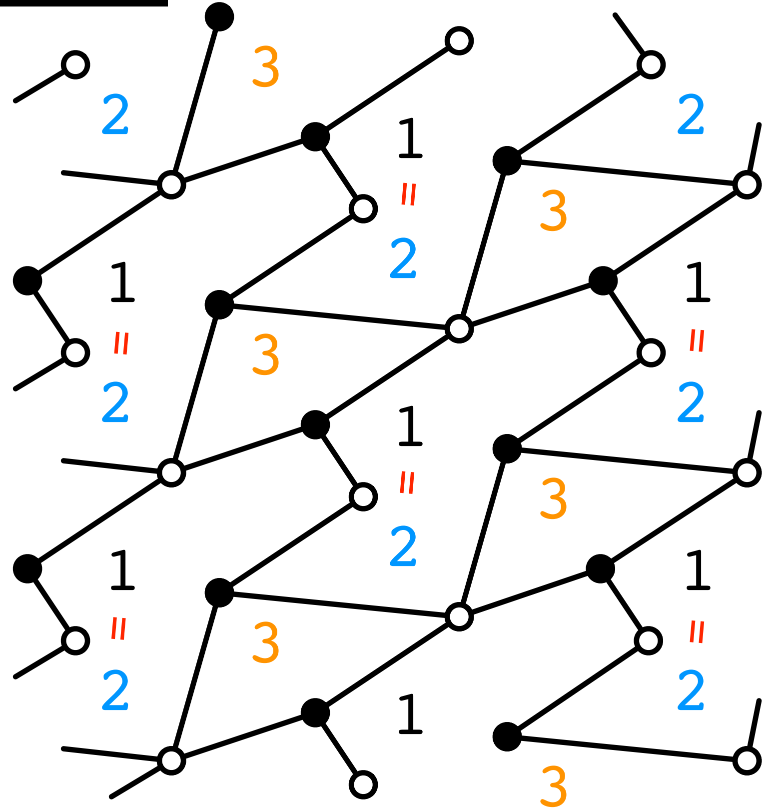


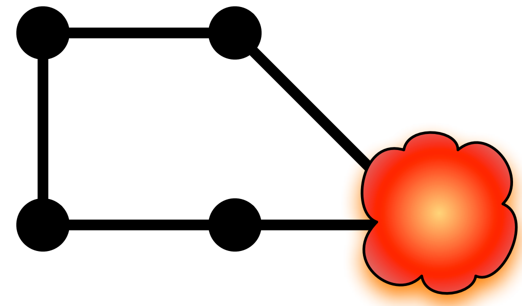
$$W = X_{13}X_{32}X_{23}Y_{31} + Y_{21}\phi X_{12} - X_{13}Y_{31}\phi - X_{12}X_{23}X_{32}Y_{21}$$

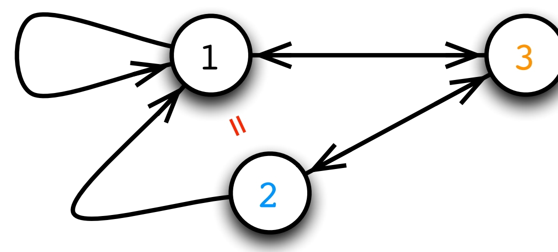
we can Higgs another bifundamental field

$$g(t;\mathcal{M}^{mes}) = \frac{1-t^6}{(1-t^2)^2(1-t^3)^2}$$

Higgsing





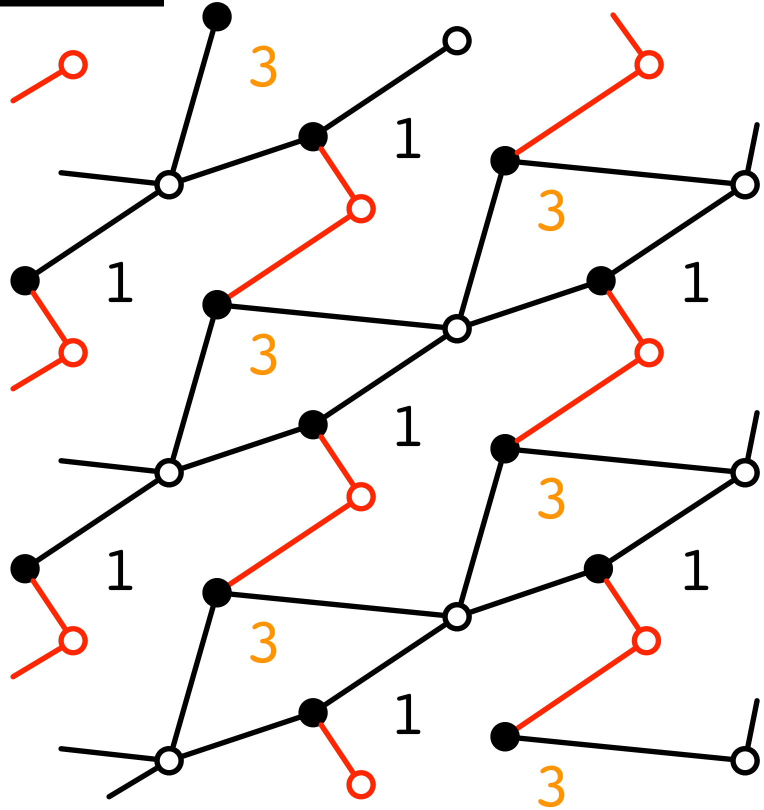


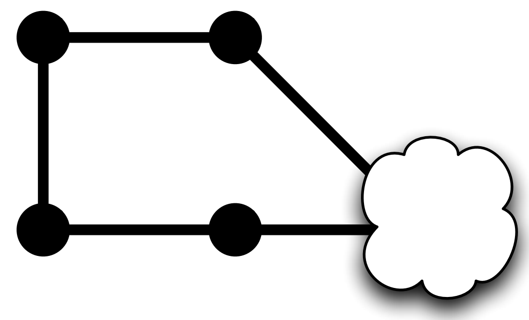
$$W = X_{13}X_{32}X_{23}Y_{31} + Y_{21}\phi - X_{13}Y_{31}\phi - X_{23}X_{32}Y_{21}$$

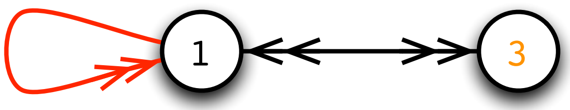
we can Higgs another bifundamental field

$$g(t;\mathcal{M}^{mes}) = \frac{1-t^6}{(1-t^2)^2(1-t^3)^2}$$

Higgsing







$$W = X_{13}X_{31}Y_{13}Y_{31} + \phi_2\phi_1$$
$$-X_{13}Y_{31}\phi_1 - Y_{13}X_{31}\phi_2$$

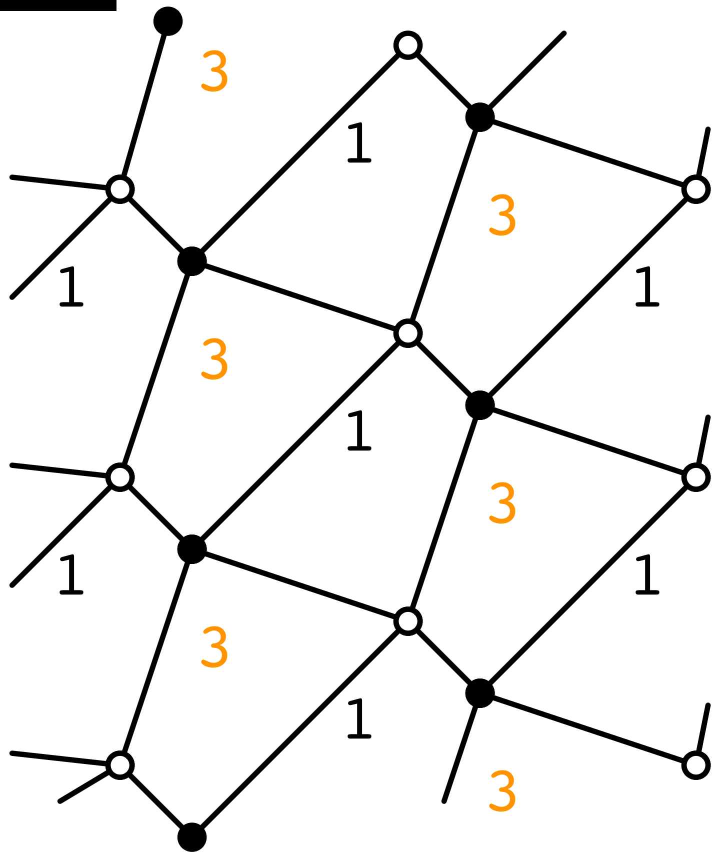
$$g(t;\mathcal{M}^{mes}) = \frac{1-t^6}{(1-t^2)^2(1-t^3)^2}$$

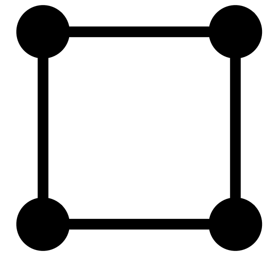
we can Higgs another bifundamental field

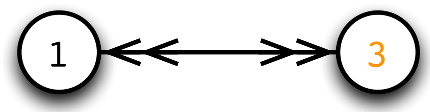
Higgsing

Conifold $\mathcal{C}$

Klebanov, Witten 1998







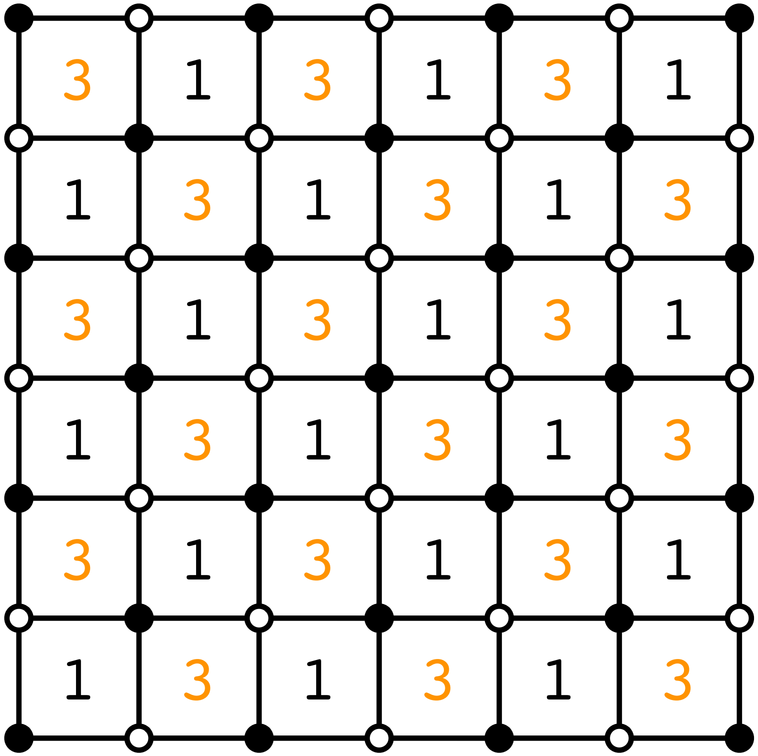
$$W = X_{13} X_{31} Y_{13} Y_{31} - X_{13} Y_{31} Y_{13} X_{31}$$

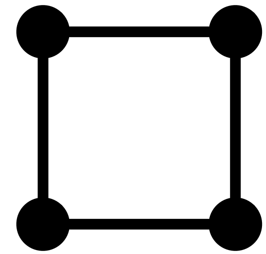
$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty} [n][n] t^{2n}$$

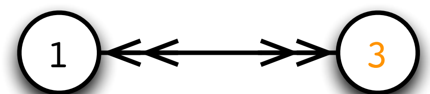
Higgsing

Conifold $\mathcal{C}$

Klebanov, Witten 1998





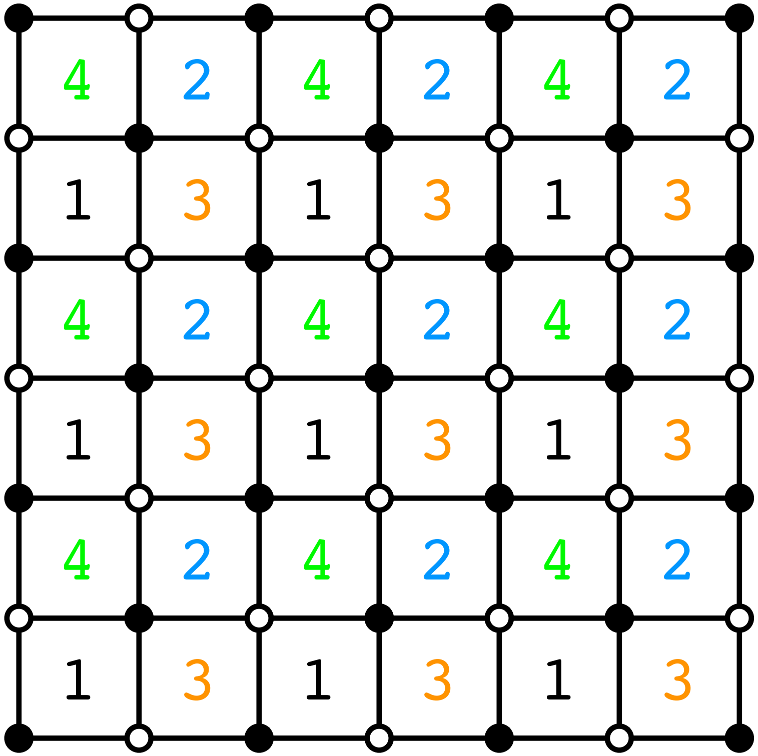


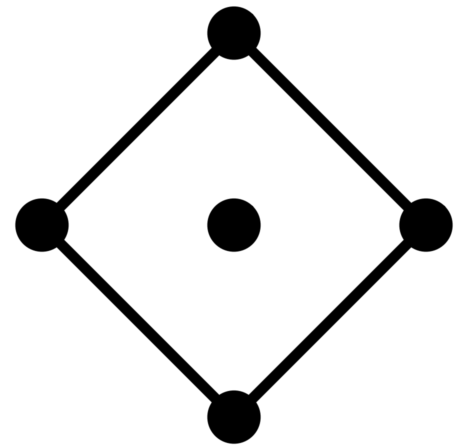
$$W = X_{13}X_{31}Y_{13}Y_{31} - X_{13}Y_{31}Y_{13}X_{31}$$

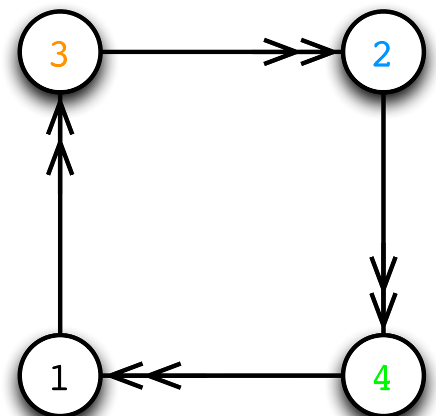
$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[n][n]t^{2n}$$

the conifold theory has a square brane tiling with valence 4 nodes

Orbifolding $\mathcal{C}/\mathbb{Z}_2\ ,\ F_0$







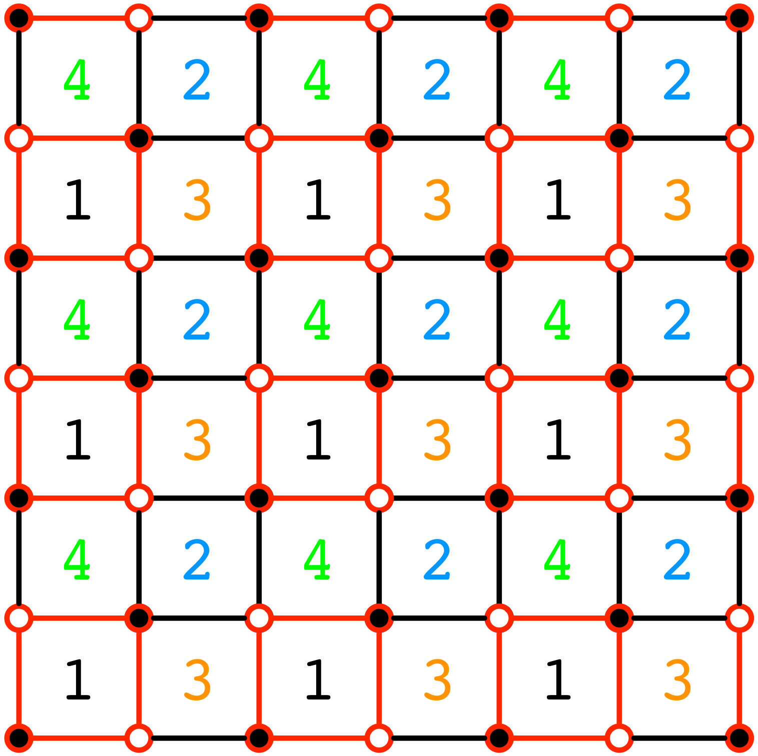
$$W = Y_{13}X_{32}X_{24}Y_{41} + Y_{24}X_{41}X_{13}Y_{32}$$
$$-X_{13}X_{32}Y_{24}Y_{41} - X_{24}X_{41}Y_{13}Y_{32}$$

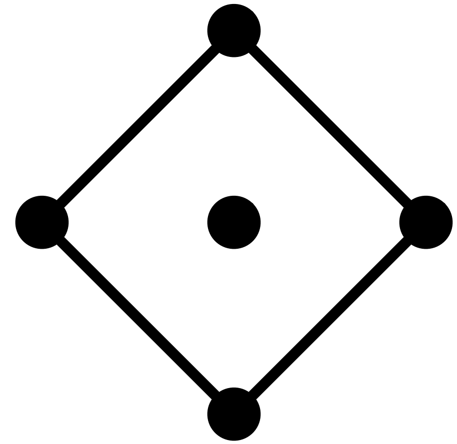
$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

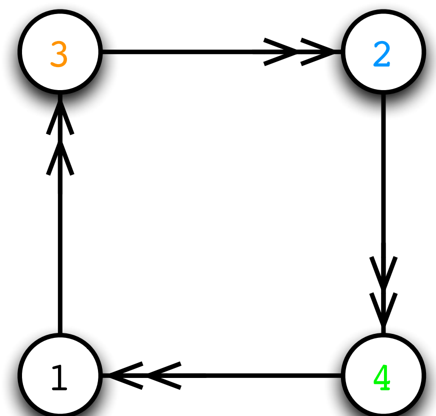
Abelian orbifolding of the conifold gives the zeroth Hirzebruch surface model

Toric (Seiberg) Duality

$\mathcal{C}/\mathbb{Z}_2$, F_0







$$W = Y_{13}X_{32}X_{24}Y_{41} + Y_{24}X_{41}X_{13}Y_{32} - X_{13}X_{32}Y_{24}Y_{41} - X_{24}X_{41}Y_{13}Y_{32}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

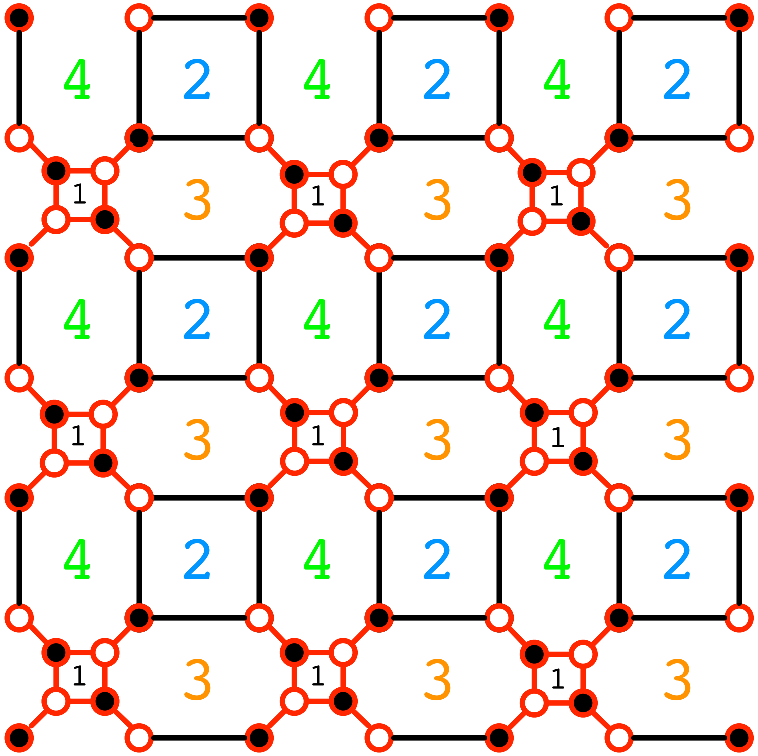
a local mutation of a square face of the tiling leaves the mesonic moduli space invariant - Toric (Seiberg) duality

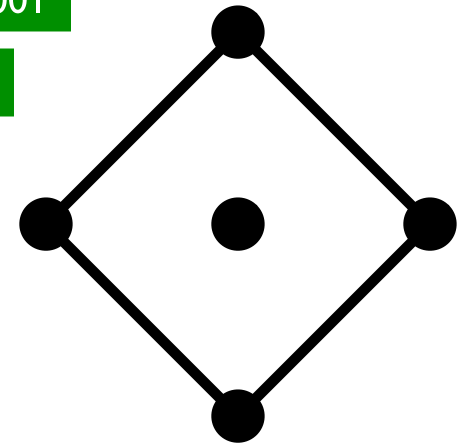
Toric (Seiberg) Duality

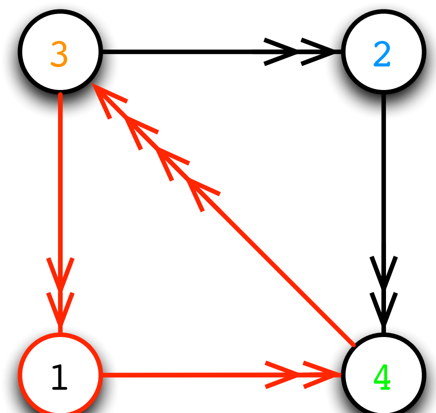
F_0 (II)

Beasley, Plesser 2001

Feng, Hanany, He, Uranga 2001







$$W = -Y_{31}Y_{14}M_{43} - X_{31}X_{14}M'_{43} + X_{32}X_{24}M_{43} + Y_{32}Y_{24}M'_{43}$$
$$+X_{31}Y_{14}M_{43} + Y_{31}X_{14}M'_{43} - X_{32}Y_{24}M_{43} - Y_{32}X_{24}M'_{43}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

a local mutation of a square face of the tiling leaves the mesonic moduli space invariant - Toric (Seiberg) duality

Toric (Seiberg) Duality

F_0 (II)

Beasley, Plesser 2001

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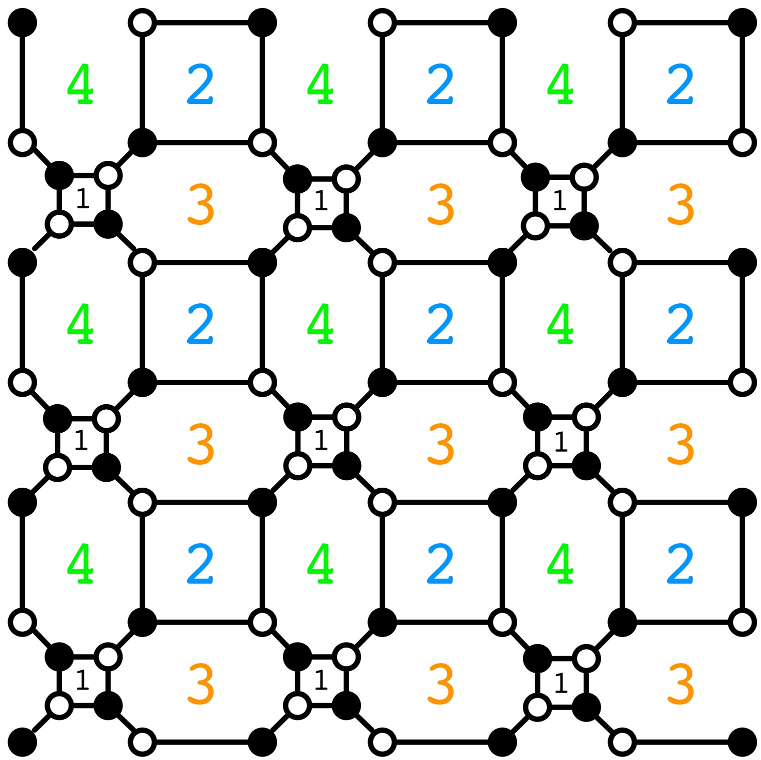
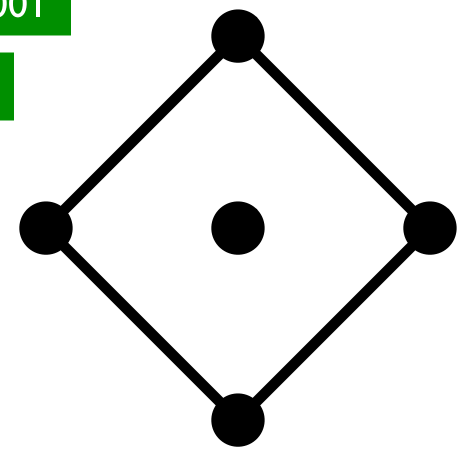
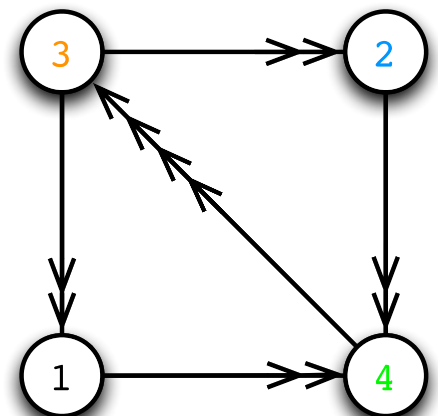






$$W = -Y_{31}Y_{14}M_{43} - X_{31}X_{14}M'_{43} + X_{32}X_{24}M_{43} + Y_{32}Y_{24}M'_{43}$$
$$+X_{31}Y_{14}M_{43} + Y_{31}X_{14}M'_{43} - X_{32}Y_{24}M_{43} - Y_{32}X_{24}M'_{43}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

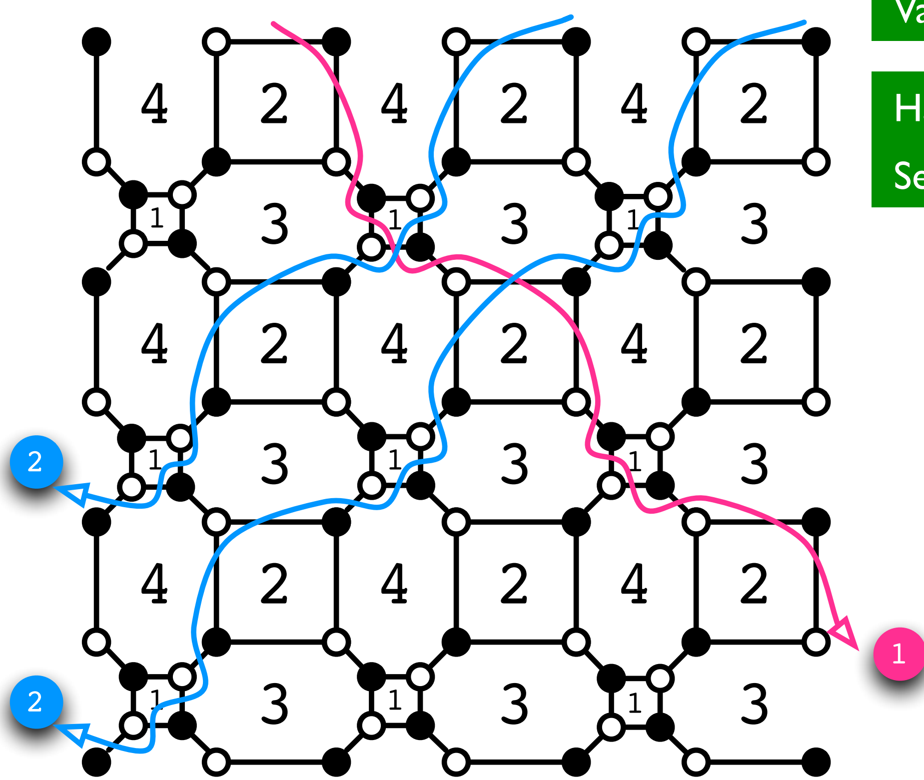
a local mutation of a square face of the tiling leaves the mesonic moduli space invariant
- Toric (Seiberg) duality

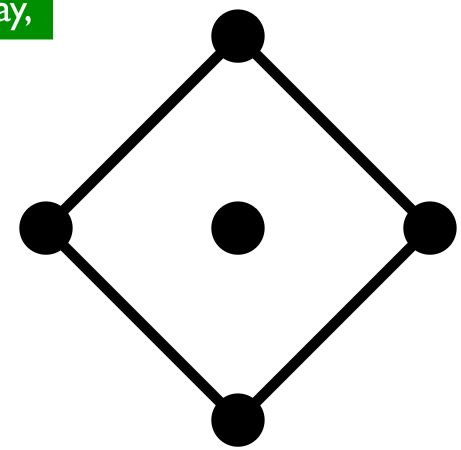
Zig-Zag Paths

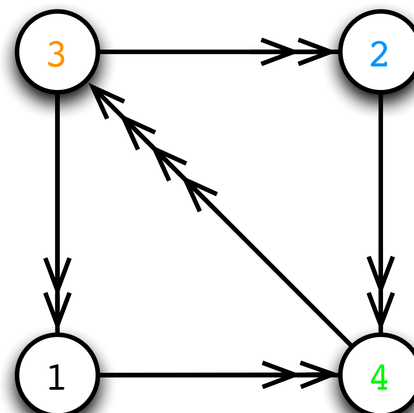
F_0 (II)

Feng, He, Kennaway, Vafa 2008

Hanany, Seong 2012







$$W = -Y_{31}Y_{14}M_{43} - X_{31}X_{14}M'_{43} + X_{32}X_{24}M_{43} + Y_{32}Y_{24}M'_{43}$$
$$+X_{31}Y_{14}M_{43} + Y_{31}X_{14}M'_{43} - X_{32}Y_{24}M_{43} - Y_{32}X_{24}M'_{43}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

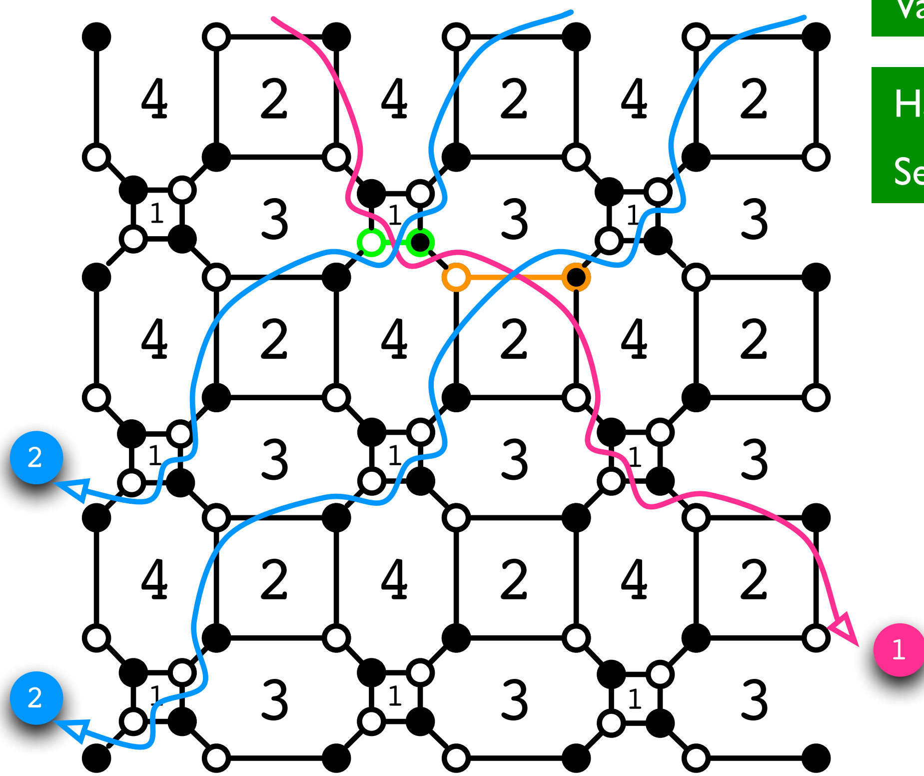
the brane tiling exhibits closed paths on the 2-torus known as zig-zag paths

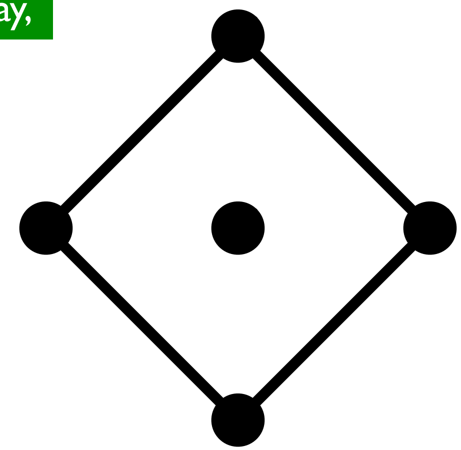
Zig-Zag Paths

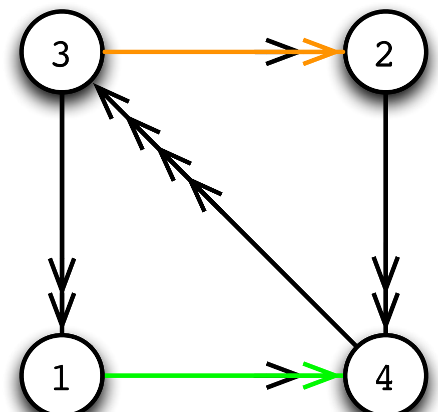
$F_0\ (II)$

Feng, He, Kennaway, Vafa 2008

Hanany, Seong 2012







$$W = -Y_{31}Y_{14}M_{43} - X_{31}X_{14}M'_{43} + X_{32}X_{24}M_{43} + Y_{32}Y_{24}M'_{43}$$
$$+X_{31}Y_{14}M_{43} + Y_{31}X_{14}M'_{43} - X_{32}Y_{24}M_{43} - Y_{32}X_{24}M'_{43}$$

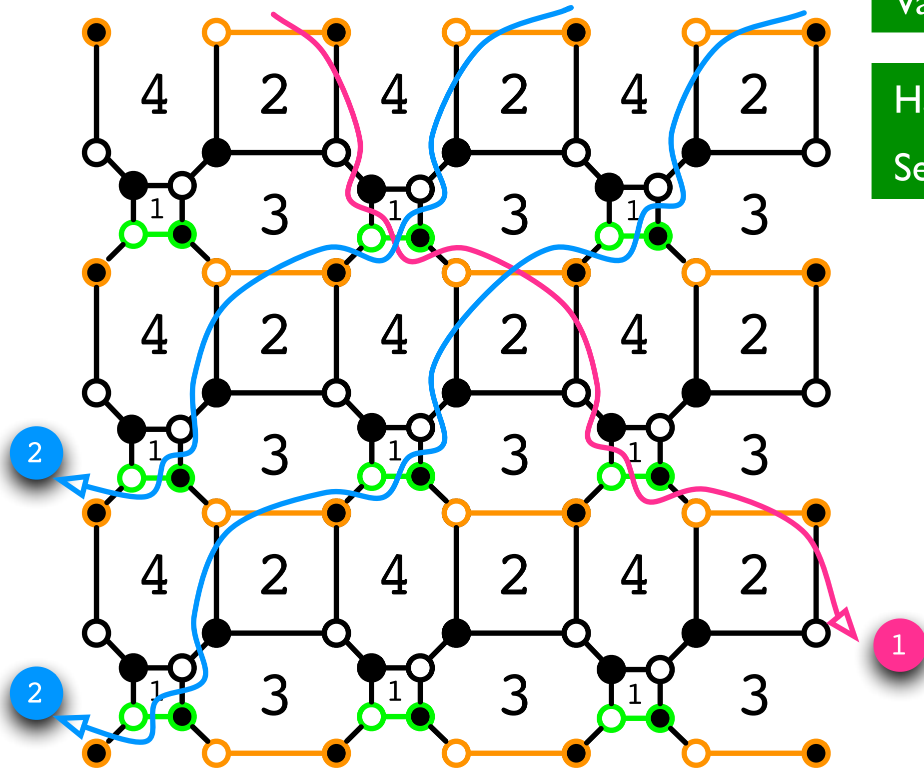
$$g(t; \mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

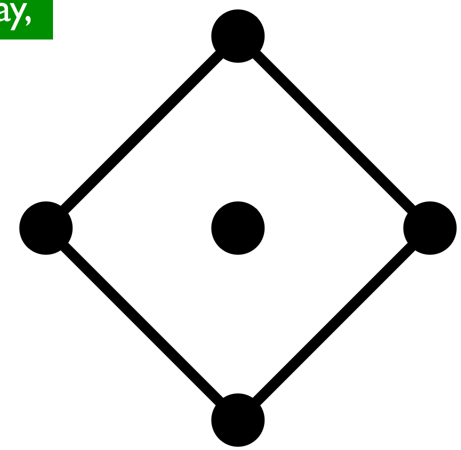
zig-zag paths intersect in tiling edges corresponding to quiver fields

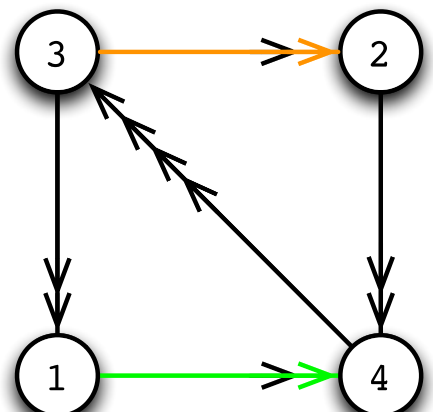
Specular Duality (untwisting)

Feng, He, Kennaway, Vafa 2008

Hanany, Seong 2012







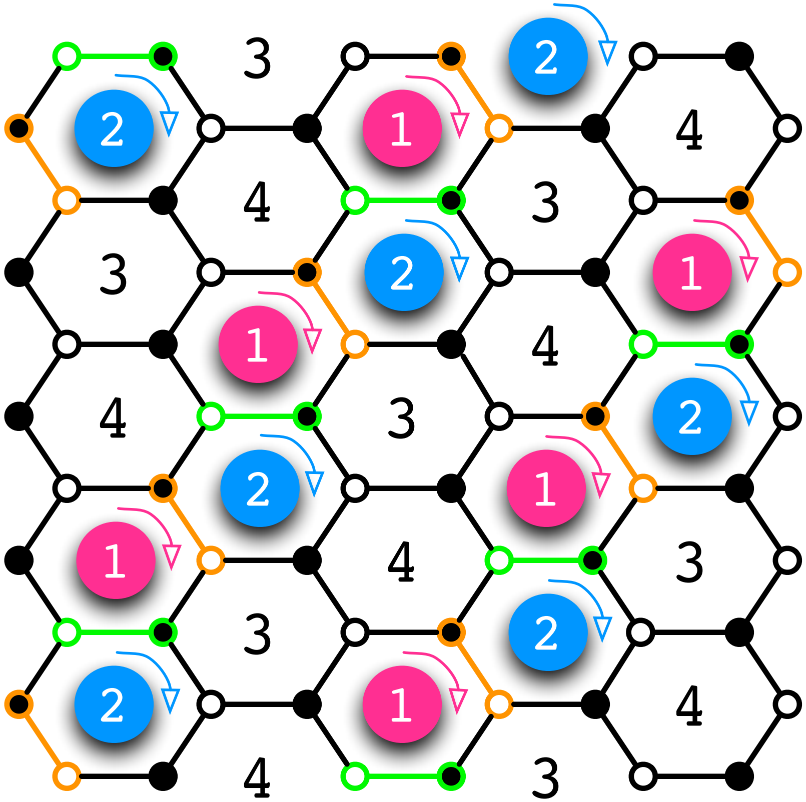
$$W = -Y_{31}Y_{14}M_{43} - X_{31}X_{14}M'_{43} + X_{32}X_{24}M_{43} + Y_{32}Y_{24}M'_{43}$$
$$+X_{31}Y_{14}M_{43} + Y_{31}X_{14}M'_{43} - X_{32}Y_{24}M_{43} - Y_{32}X_{24}M'_{43}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

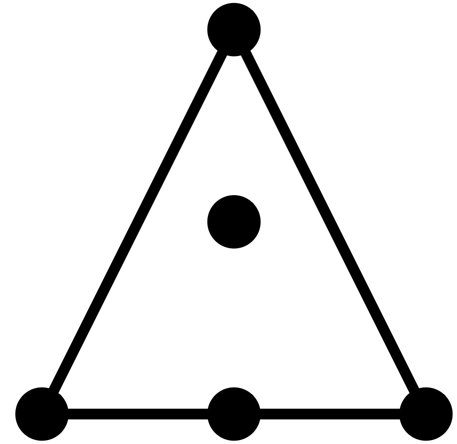
by a mutation known as the untwisting map, the zig-zag paths can be mapped to faces of a new brane tiling

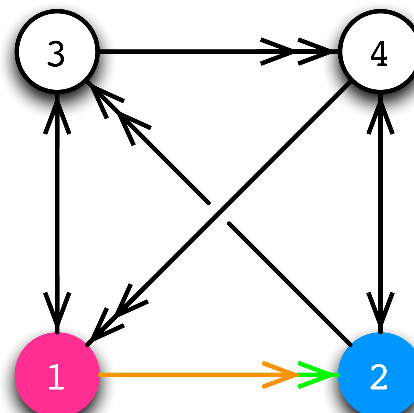
Specular Duality (untwisting)

$\mathbb{C}^3/\mathbb{Z}_4$



zig-zag paths become gauge groups





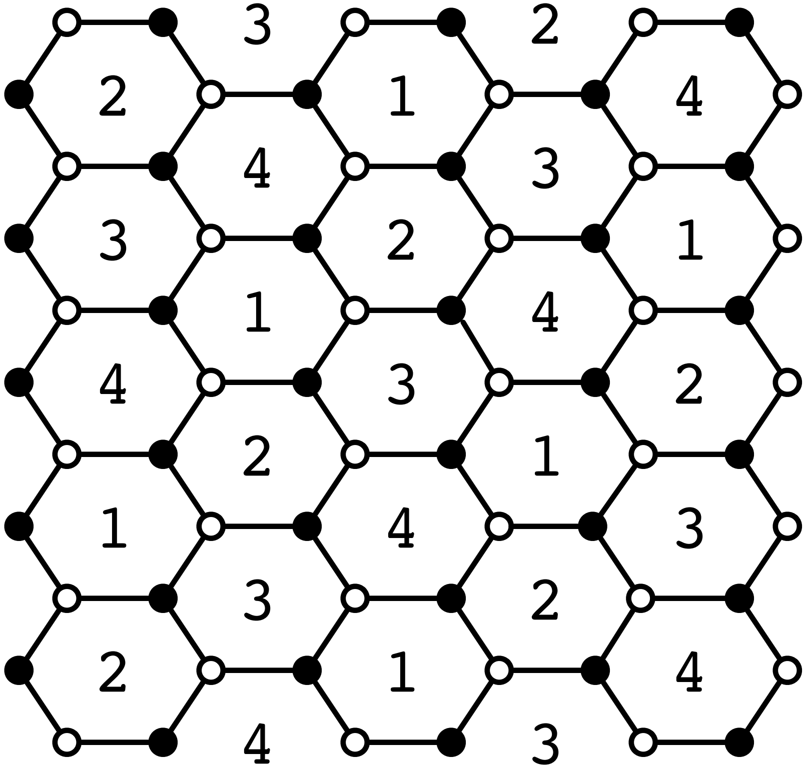
$$W = X_{12}Y_{23}X_{31} + X_{23}Y_{34}X_{42} + X_{34}Y_{41}X_{13} + X_{41}Y_{12}X_{24}$$
$$-X_{31}Y_{12}X_{23} - X_{42}Y_{23}X_{34} - X_{13}Y_{34}X_{41} - X_{24}Y_{41}X_{12}$$

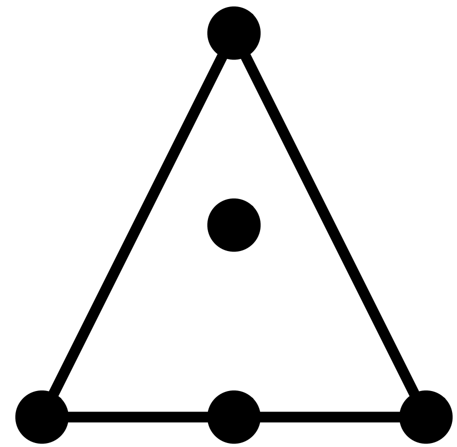
$$g(t;\mathcal{M}^{mes}) = \sum_{n_i=0}^{\infty}([2n_1]t^{3n_1+2n_2} + [2n_1+4n_2+4]t^{3n_1+4n_2+4})$$

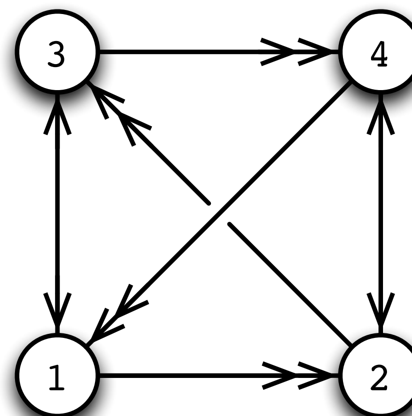
Hanany, Seong 2012

Specular Duality (untwisting)

$\mathbb{C}^3/\mathbb{Z}_4$





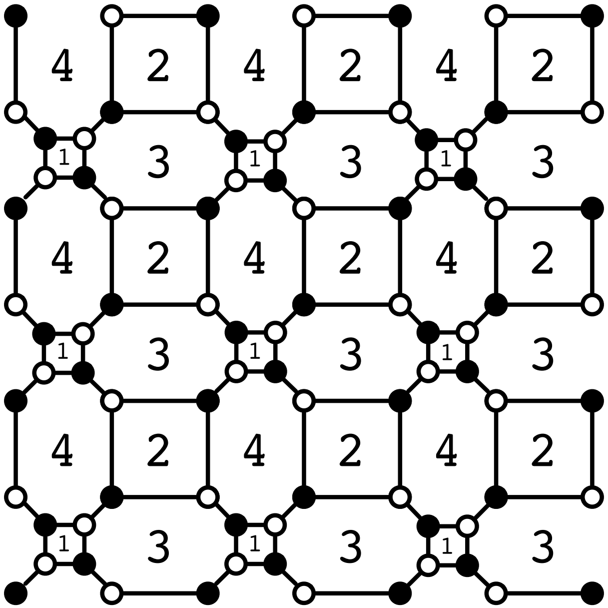


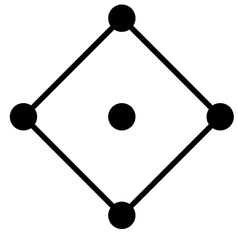
$$W = X_{12}Y_{23}X_{31} + X_{23}Y_{34}X_{42} + X_{34}Y_{41}X_{13} + X_{41}Y_{12}X_{24}$$
$$-X_{31}Y_{12}X_{23} - X_{42}Y_{23}X_{34} - X_{13}Y_{34}X_{41} - X_{24}Y_{41}X_{12}$$

$$g(t;\mathcal{M}^{mes}) = \sum_{n_i=0}^{\infty}([2n_1]t^{3n_1+2n_2} + [2n_1+4n_2+4]t^{3n_1+4n_2+4})$$

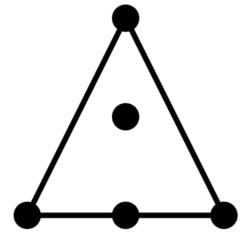
Hanany, Seong 2012

Specular Duality (untwisting)

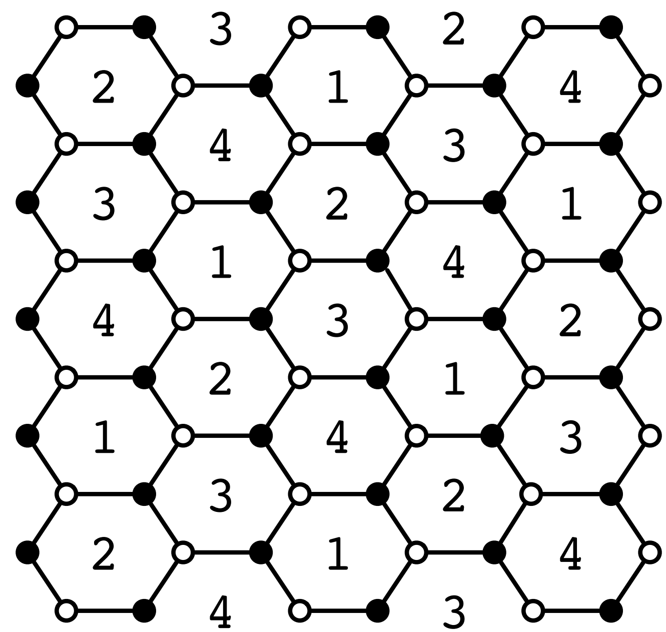




$F_0\ (II)$



$\mathbb{C}^3/\mathbb{Z}_4$



$$g(t;\mathcal{M}^{mes}) = \sum_{n=0}^{\infty}[2n][2n]t^{4n}$$

the mesonic moduli spaces are different

$$g(t;\mathcal{M}^{mes}) = \sum_{n_i}^{\infty}\left([2n_1]t^{3n_1+2n_2} + [2n_1+4n_2+4]t^{3n_1+4n_2+4}\right)$$

the spectrum of both mesonic and baryonic GIOs (master space) is the same – specular duality

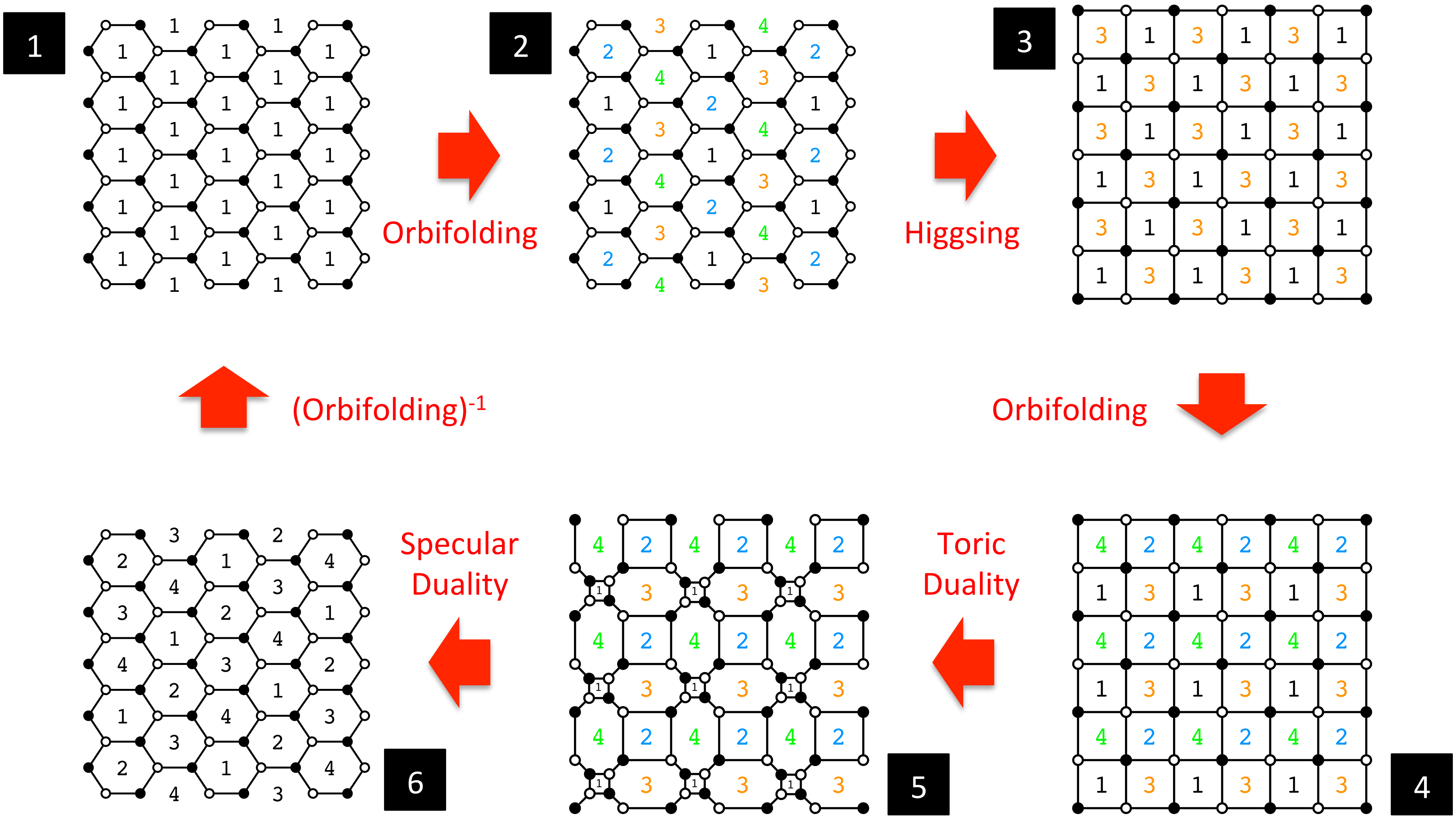
$$g(t;\mathcal{F}^\flat) = \sum_{n_i=0}^{\infty}[n_1+n_2][n_2+n_3][n_1+n_3]t^{n_1+n_2+2n_3}$$

Forcella, Hanany, He, Zaffaroni 2008

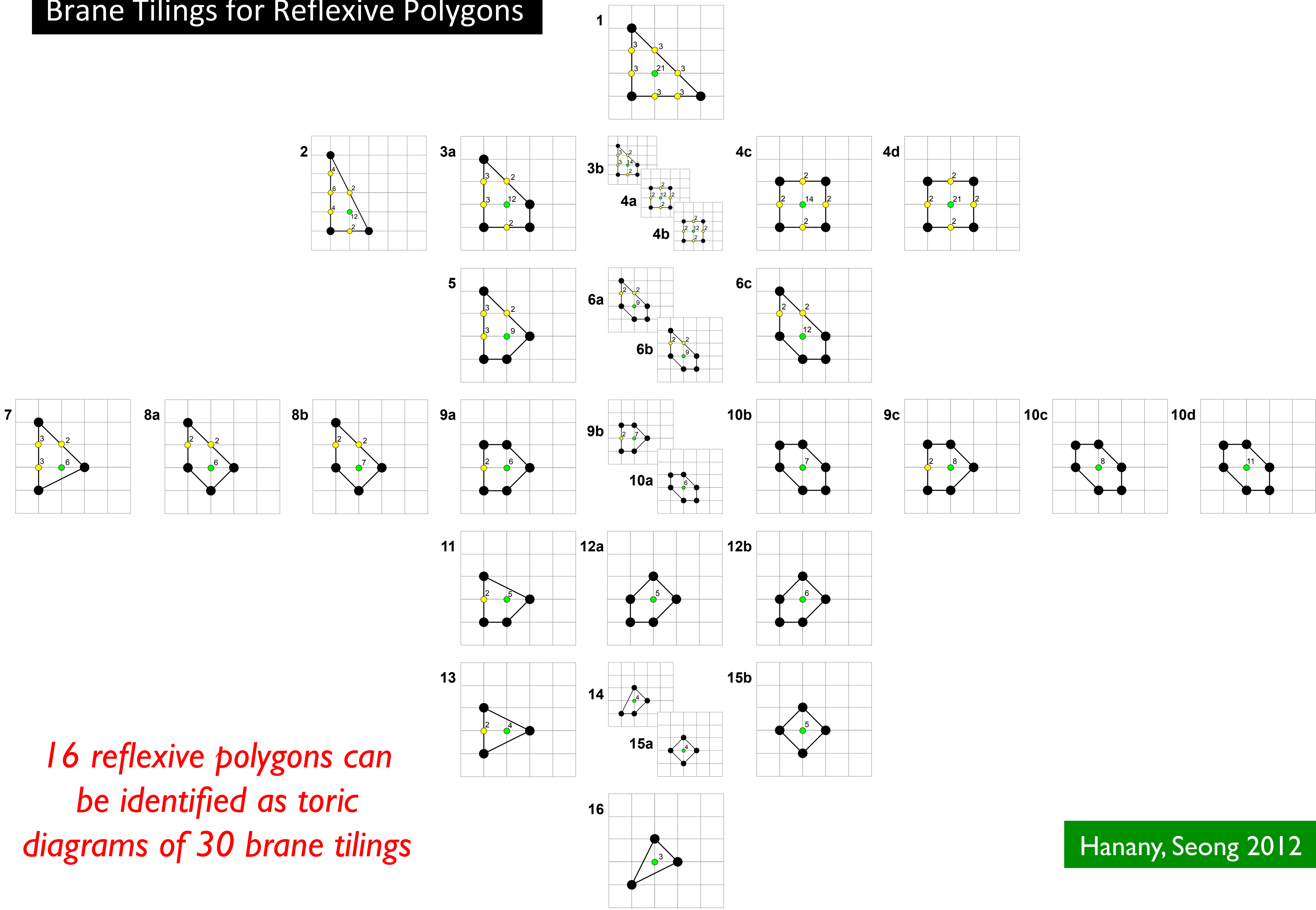
Hanany, Seong 2012

Brane Tiling Cycle

Orbifolding and Higgsing generates an infinite class of brane tilings



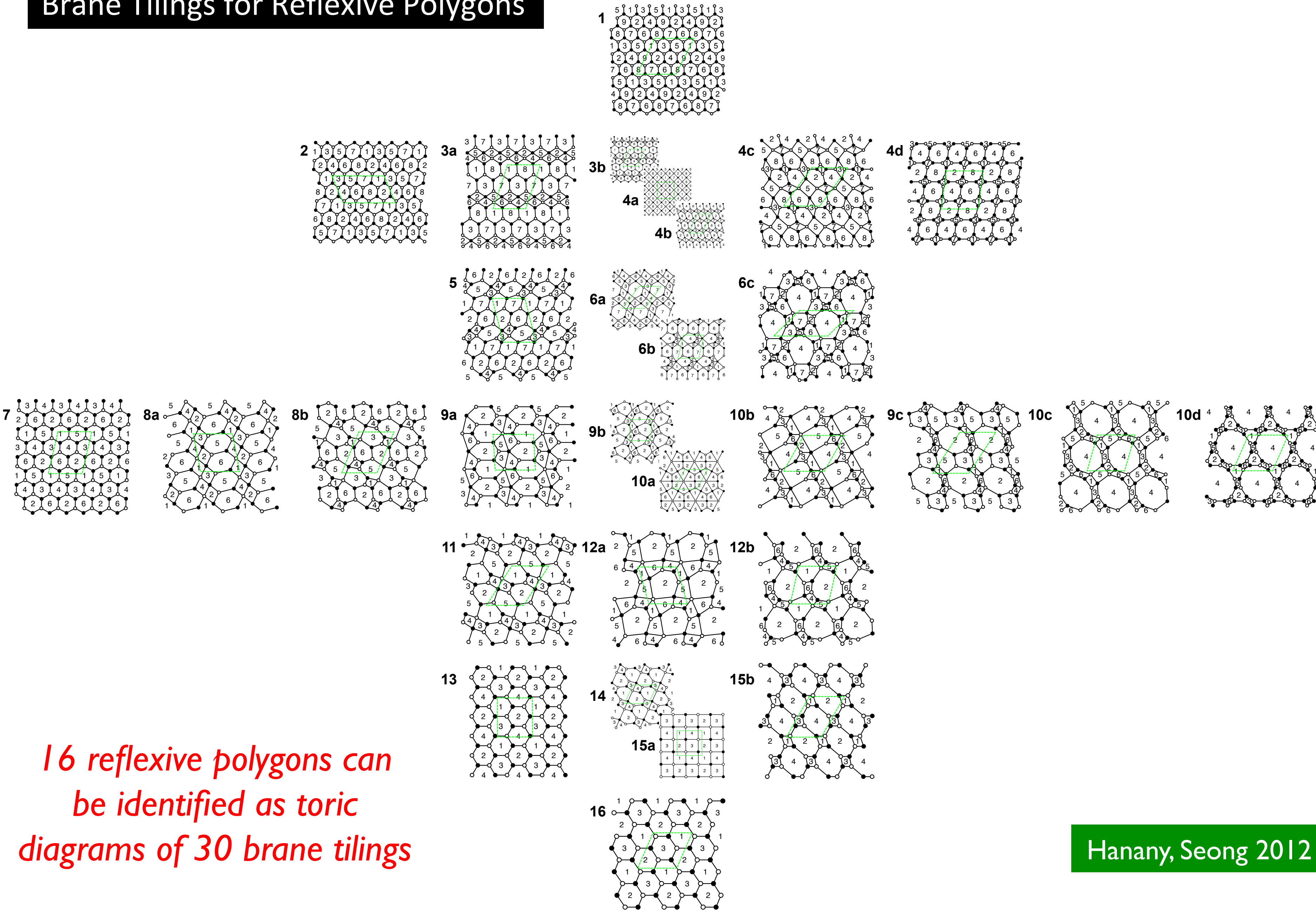
Brane Tilings for Reflexive Polygons



16 reflexive polygons can be identified as toric diagrams of 30 brane tilings

Hanany, Seong 2012

Brane Tilings for Reflexive Polygons

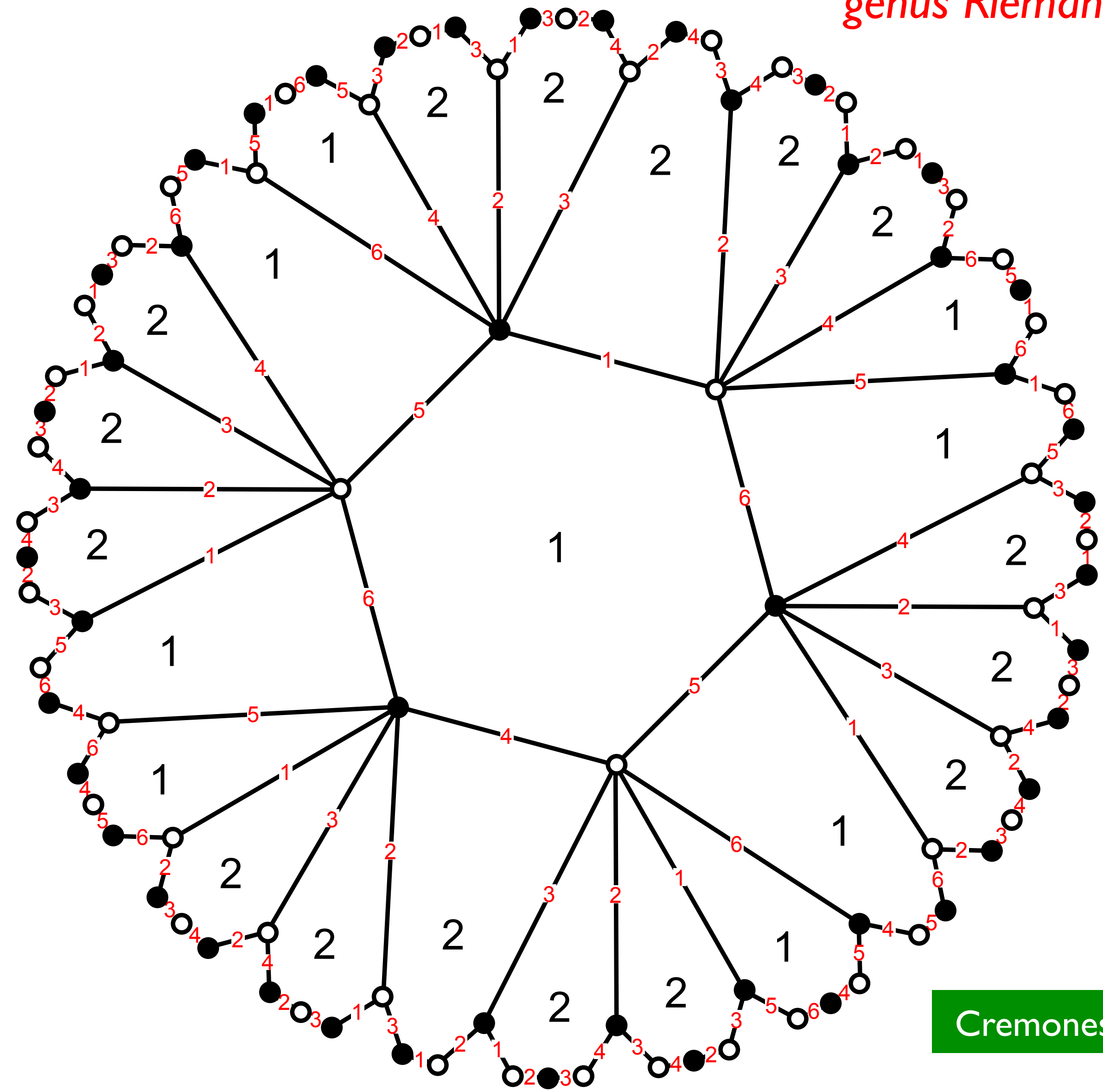


16 reflexive polygons can be identified as toric diagrams of 30 brane tilings

Hanany, Seong 2012

Brane Tilings on a genus 2 Riemann surface

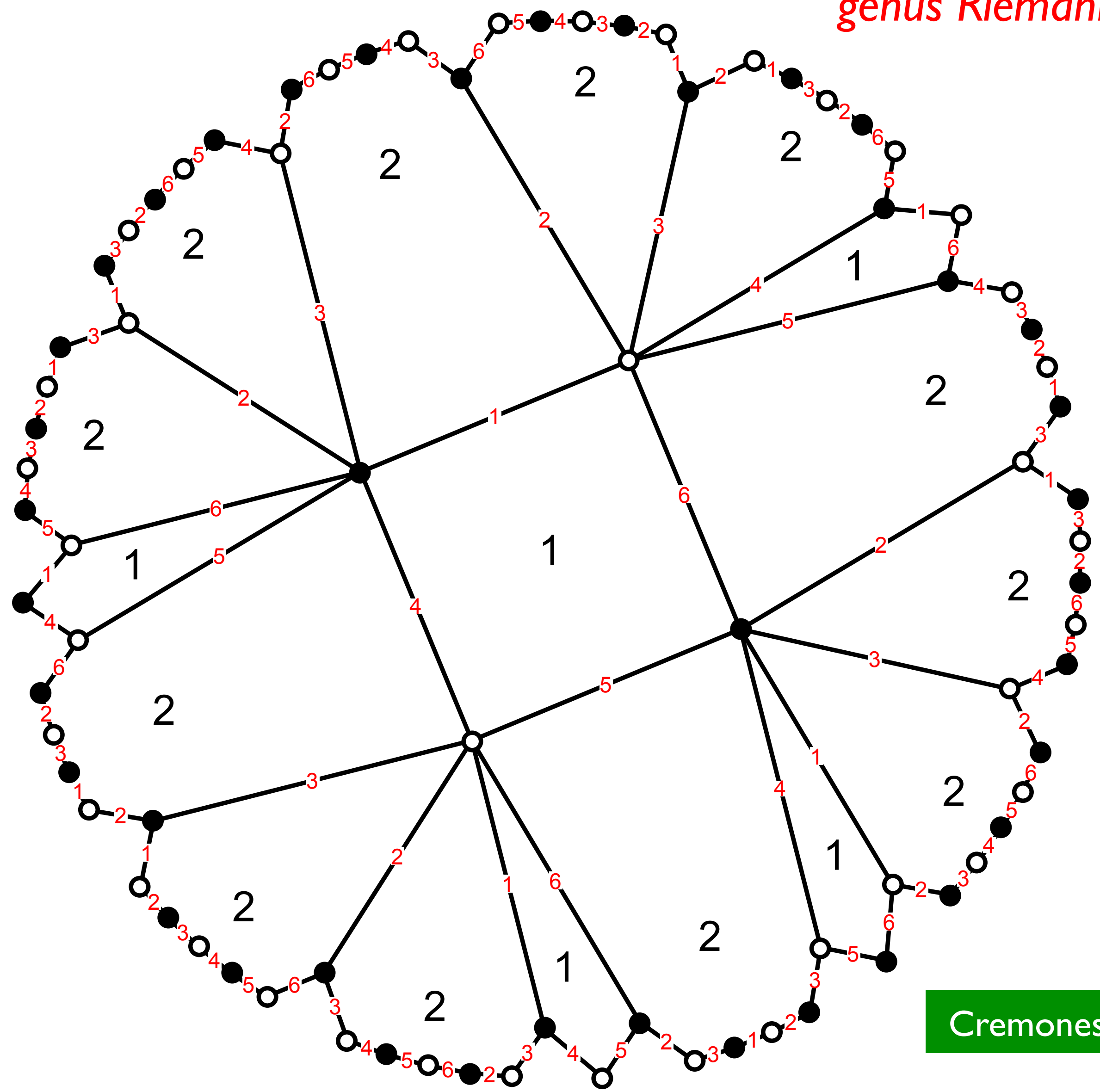
specular duality can be used to generate brane tilings on higher genus Riemann surfaces



Cremonesi, Hanany, Seong 2013

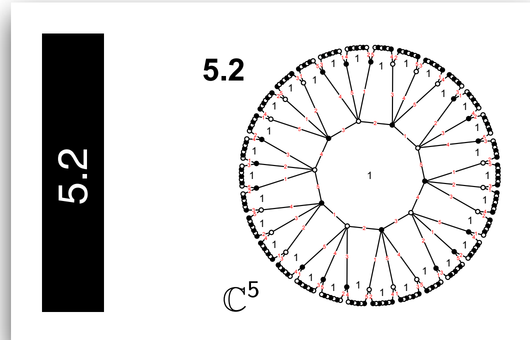
Brane Tilings on a genus 2 Riemann surface

specular duality can be used to generate brane tilings on higher genus Riemann surfaces

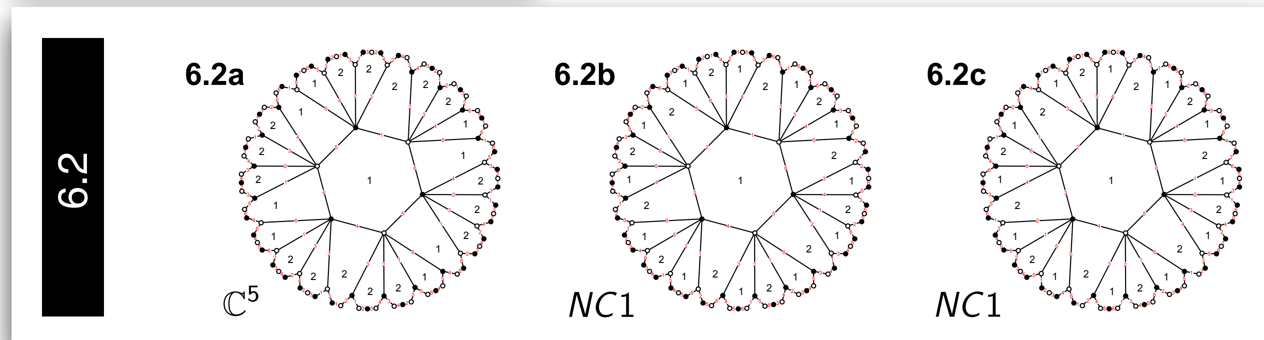


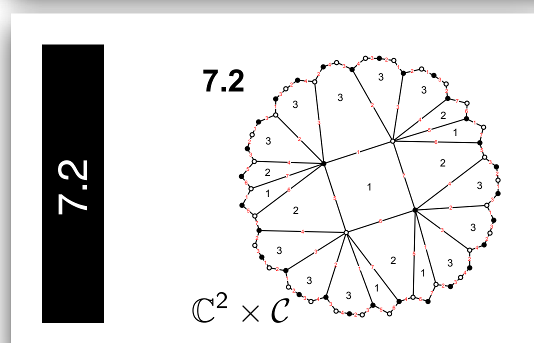
Cremonesi, Hanany, Seong 2013

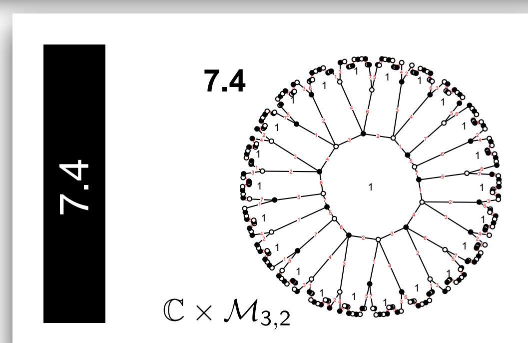
Brane Tilings on a genus 2 Riemann surface



an unexplored class of supersymmetric quiver gauge theories is represented by brane tilings on Riemann surfaces

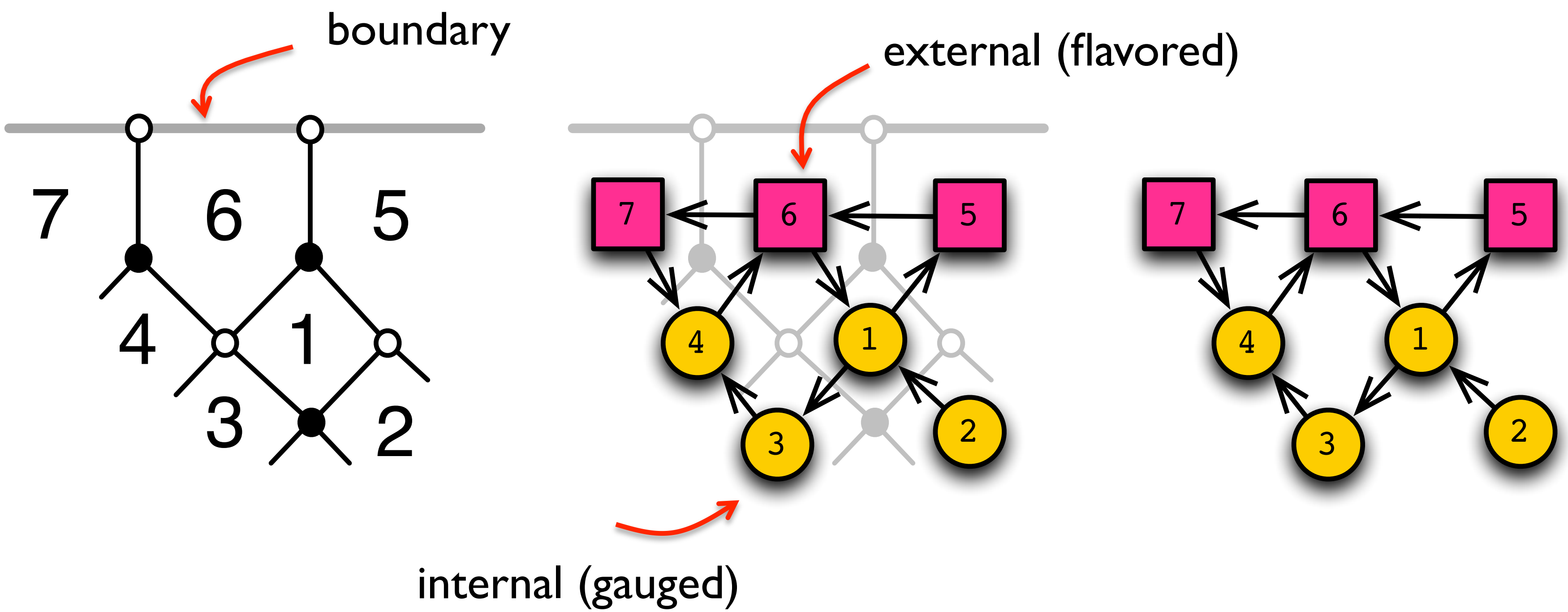






Cremonesi, Hanany, Seong 2013

Brane Tilings on Riemann surfaces with boundaries

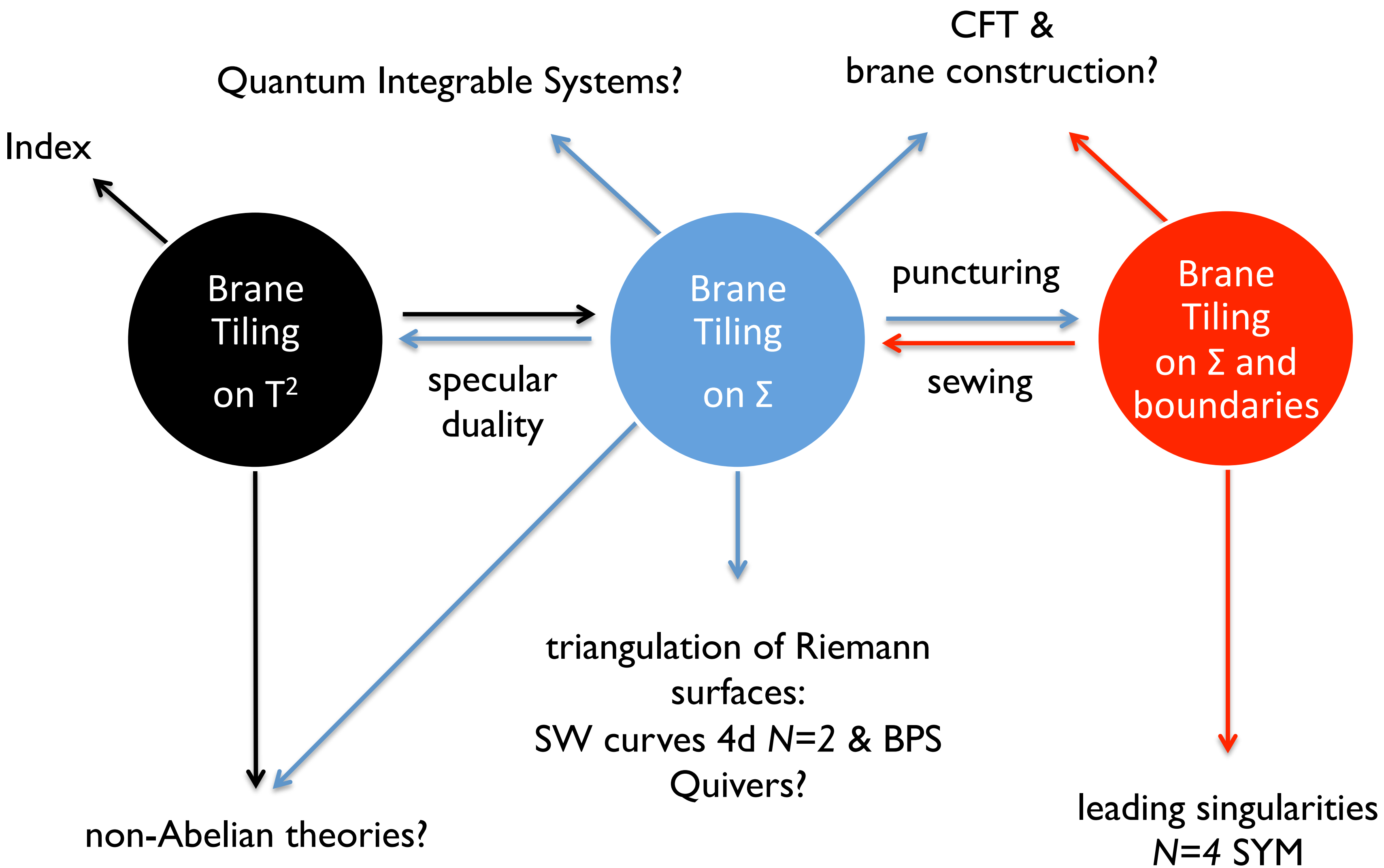


specular duality can be used to generate brane tilings on higher genus Riemann surfaces

Franco, Galloni, Seong 2012

Yamazaki, Xie 2012

Open Questions & Future Directions



Thank You