PARA-BOSONIC STRINGS AND SPACE-TIME NON-COMMUTATIVITY

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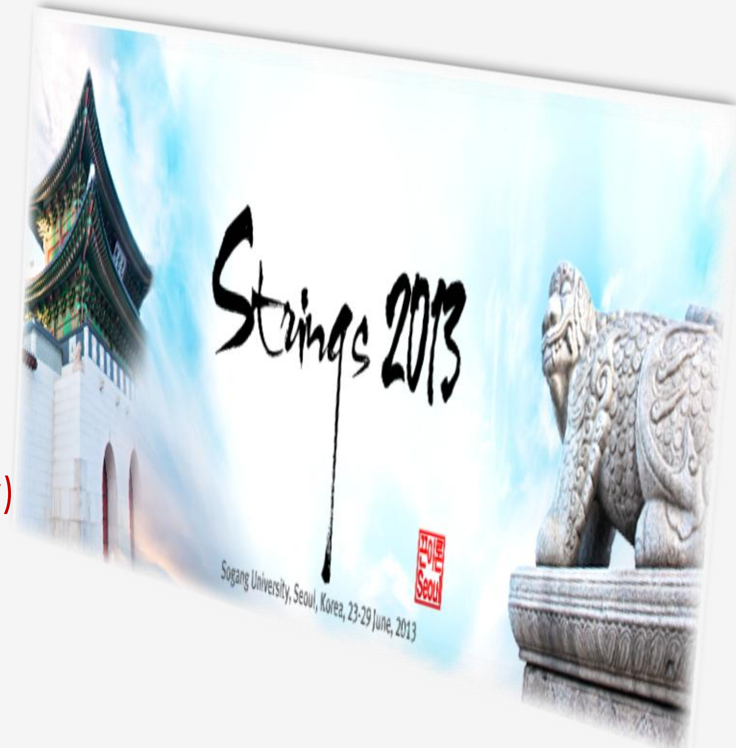

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Para-quantum formalism and Bosonic string theory:

Paraquantization (Green 1953)

- Generalization of the **B.E** and **F.D** Statistics
  - Parabosons Sp(2f,R) Parafermions SO(2f)
- Green decomposition:
  - Trilinear commutation relations. ⟷ Bilinear commutation relations Of an anomalous case. [1]

Para-Strings

- Provides other possibilities of critical dimensions for Bosonic and Fermionic theories.
- Parabosonic case: $D-2=\frac{24}{Q}$. [2,3,4,5]

The Model:

$$\left[X^I(\tau,\sigma),\left[X^J(\tau,\sigma'),X^K(\tau,\sigma'')\right]_+\right] = 2i\left\{\theta^{IJ}(\sigma-\sigma')X^K(\tau,\sigma'')+\theta^{IK}(\sigma-\sigma'')X^J(\tau,\sigma')\right\}$$

$$\left[X^I(\tau,\sigma),\left[\Pi^J(\tau,\sigma'),X^K(\tau,\sigma'')\right]_+\right] = 2i\left\{\eta^{IJ}\delta(\sigma-\sigma')X^K(\tau,\sigma'')+\theta^{IK}(\sigma-\sigma'')\Pi^J(\tau,\sigma')\right\}$$

$$\left[X^I(\tau,\sigma),\left[\Pi^J(\tau,\sigma'),\Pi^K(\tau,\sigma'')\right]_+\right] = 2i\left\{\eta^{IJ}\delta(\sigma-\sigma')\Pi^K(\tau,\sigma'')+\eta^{IK}\delta(\sigma-\sigma'')\Pi^J(\tau,\sigma')\right\}$$

$$\left[\Pi^I(\tau,\sigma),\left[X^J(\tau,\sigma'),\Pi^K(\tau,\sigma'')\right]_+\right] = -2i\left\{\eta^{IJ}\delta(\sigma'-\sigma)\Pi^K(\tau,\sigma'')+\gamma^{IK}(\sigma-\sigma'')X^J(\tau,\sigma')\right\}$$

$$\left[\Pi^I(\tau,\sigma),\left[\Pi^J(\tau,\sigma'),\Pi^K(\tau,\sigma'')\right]_+\right] = 2i\{\gamma^{IJ}(\sigma-\sigma')\Pi^K(\tau,\sigma'')+\gamma^{IK}(\sigma-\sigma'')\Pi^J(\tau,\sigma')\}$$

$$[X^I(\tau,\sigma),[X^J(\tau,\sigma'),A]_+] = 2i\theta^{IJ}(\sigma-\sigma')A.$$

$$[X^I(\tau,\sigma),[\Pi^J(\tau,\sigma'),A]_+] = 2i\eta^{IJ}\delta(\sigma-\sigma')A.$$

$$[x^-,[p^+,B]_+] = 2iB.$$

$$[x^-,[p^+,p^+]_+] = 4ip^+.$$

With $A = x^-, p^+$ and $B = x^-, X^I(\tau,\sigma), \Pi^I(\tau,\sigma)$.

The non-commutativity parameters supposed to be 2-tensors operators to maintain Lorentz invariance [6], are developped in a Fourier series expansion[7]: $\theta^{IJ}(\sigma-\sigma') = \sum_{k=-\infty}^{+\infty} \theta_k^{IJ} \exp ik(\sigma-\sigma')$ $\quad\gamma^{IJ}(\sigma-\sigma'') = \sum_{k=-\infty}^{+\infty} \gamma_k^{IJ} \exp ik(\sigma-\sigma')$

↓ ↓

$$\left[i\theta^{IJ}(\sigma-\sigma')\right]^+ = [i\theta^{JI}(\sigma'-\sigma)]. \qquad \left[i\gamma^{IJ}(\sigma-\sigma')\right]^+ = [i\gamma^{JI}(\sigma'-\sigma)].$$

Oscillators modes and Mass center variables trilinear relations:

Oscillators modes l,m,n ≠ 0 :

$$\left[\tilde{\alpha}^I_m, \left[\tilde{\alpha}^J_n, \tilde{\alpha}^K_l\right]_+\right] = 2\left\{(m\eta^{IJ} + i\frac{n^2}{2\alpha'}\theta^{IJ}_n + i\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IJ}_n)\delta_{m+n,0}\tilde{\alpha}^K_l + (m\eta^{IK} + i\frac{l^2}{2\alpha'}\theta^{IK}_l + i\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IK}_l)\delta_{m+l,0}\tilde{\alpha}^J_n\right\}$$

$$\left[\alpha^I_m, \left[\alpha^J_n, \alpha^K_l\right]_+\right] = 2\left\{(m\eta^{IJ} + i\frac{n^2}{2\alpha'}\theta^{IJ}_{-n} + i\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IJ}_{-n})\delta_{m+n,0}\alpha^K_l + (m\eta^{IK} + i\frac{l^2}{2\alpha'}\theta^{IK}_{-l} + i\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IK}_{-l})\delta_{m+l,0}\alpha^J_n\right\}$$

$$\left[\tilde{\alpha}^I_m, \left[\alpha^J_n, \tilde{\alpha}^K_l\right]_+\right] = 2(m\eta^{IK} + i\frac{l^2}{2\alpha'}\theta^{IK}_l + i\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IK}_l)\alpha^J_n\delta_{m+l,0}.$$

Mass center variables:

$$\left[p^I, \left[p^J, p^K\right]_+\right] = \frac{2i}{2\alpha'}\frac{2}{\alpha'}(2\pi\alpha')^2\{\gamma^{IJ}_0 p^K + \gamma^{IK}_0 p^J\}.$$

$$\left[x^I, \left[p^J, p^K\right]_+\right] = 2i\{\eta^{IJ} - \frac{(2\pi\alpha')^2}{2\alpha'}2\tau\gamma^{IJ}_0\}p^K + 2i\{\eta^{IK} - \frac{(2\pi\alpha')^2}{2\alpha'}2\tau\gamma^{IK}_0\}p^J.$$

$$\left[x^I, \left[p^J, x^K\right]_+\right] = 2i\{\eta^{IJ} - \frac{(2\pi\alpha')^2}{2\alpha'}2\tau\gamma^{IJ}_0\}x^K + 2i\{\theta^{IK}_0 + (2\pi\alpha')^2\tau^2\gamma^{IK}_0\}p^J.$$

$$\left[x^I, \left[x^J, x^K\right]_+\right] = 2i\left\{\theta^{IJ}_0(x^K) + \theta^{IK}_0(x^J)\right\} - 2i(2\pi\alpha')^2\tau^2\{\gamma^{IJ}_0 x^K\delta_{n,0} + \gamma^{IK}_0 x^J\delta_{l,0}\}.$$

$$\left[x^I, \left[p^J, D\right]_+\right] = 2i\{\eta^{IJ} - \frac{(2\pi\alpha')^2}{2\alpha'}2\tau\gamma^{IJ}_0\}D$$

$$\left[x^I, \left[x^J, D\right]_+\right] = 2i\{\theta^{IJ}_0 + (2\pi\alpha')^2\tau^2\gamma^{IJ}_0\}D$$

$$\left[p^I, \left[p^J, D\right]_+\right] = \frac{2i}{2\alpha'}\frac{2}{\alpha'}\left\{(2\pi\alpha')^2\gamma^{IJ}_0\right\}D.$$

$$\left[x^-, \left[p^+, C\right]_+\right] = 2iC.$$

With $C = \{x^-, x^i, p^i, \alpha^I_m, \tilde{\alpha}^I_m\}$,and $D = \{x^-, p^+, \alpha^I_m, \tilde{\alpha}^I_m\}$.

Virasoro Algebra:

Symmetrized Virasoro generators:

$$L_m^{\perp} = \frac{1}{4}\sum_{p=-\infty}^{+\infty}\left[\alpha^I_{m-p},\alpha_{pI}\right]_+ \qquad \tilde{L}_m^{\perp} = \frac{1}{4}\sum_{p=-\infty}^{+\infty}\left[\tilde{\alpha}^I_{m-p},\tilde{\alpha}_{pI}\right]_+$$

Virasoro algebra:

$$\left[L_m^{\perp}, L_n^{\perp}\right] = (m-n)L_{m+n} - Q(D-2)\frac{(n^3-n)}{12}\delta_{m+n,0} + \pounds_{mn,IJ}$$

$$\left[\tilde{L}_m^{\perp}, \tilde{L}_n^{\perp}\right] = (m-n)\tilde{L}_{m+n} - Q(D-2)\frac{(n^3-n)}{12}\delta_{m+n,0} + \tilde{\pounds}_{mn,IJ}$$

$$\left[L_m^{\perp}, \tilde{L}_n^{\perp}\right] = 0$$

New anomaly term:

$$\pounds_{mn,IJ} = \sum_{p=-\infty}^{+\infty}\left\{\frac{i}{4}\frac{(p-m)^2}{2\alpha'}\theta^{IJ}_{(m-p)} + \frac{i}{4}\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IJ}_{(m-p)}\right\}\left[\alpha_{pI},\alpha_{m+n-pJ}\right]_+$$

$$\tilde{\pounds}_{mn,IJ} = \sum_{p=-\infty}^{+\infty}\left\{\frac{i}{4}\frac{(p-m)^2}{2\alpha'}\theta^{IJ}_{(p-m)} + \frac{i}{4}\frac{(2\pi\alpha')^2}{2\alpha'}\gamma^{IJ}_{p-m}\right\}\left[\tilde{\alpha}_{pI},\tilde{\alpha}_{m+n-pJ}\right]_+$$

Diagonalisation of the Mass operator:

- Before mass diagonalisation: Open string 2nd level:

$$M^2\{\alpha^J_{-2}|p^+,\vec{p}^T\rangle\} = \frac{1}{2\alpha'}\sum_{I=2}^{D-1}\left\{\begin{array}{c}(4-\frac{Q(D-2)}{12})\alpha^J_{-2} - \frac{i4}{\alpha'}\theta_2^{IJ}\alpha_{-2I} \\ -\frac{i}{\alpha'}(2\pi\alpha')^2\gamma_2^{IJ}\alpha_{-2I}\end{array}\right\}|p^+,\vec{p}^T\rangle.$$

- Diagonalisation of the θ, γ matrices:

$$D_n = \tilde{U}^{-1}i\theta_n\tilde{U} = \mu_I^{(n)}\delta_{I,J} \qquad T_n = \tilde{U}^{-1}i\gamma_n\tilde{U} = \nu_I^{(n)}\delta_{I,J}$$

$$D'_{-n} = U^{-1}i\theta_{-n}U = \mu_I^{(-n)}\delta_{I,J} \qquad T'_{-n} = U^{-1}i\gamma_{-n}U = \nu_I^{(-n)}\delta_{I,J}$$

$$[\theta_n,\gamma_n] = 0. \qquad [\theta_{-n},\gamma_{-n}] = 0.$$

$$\Downarrow$$

$$\tilde{U}^{-1}[\theta_n,\gamma_n]\tilde{U} = 0. \qquad U^{-1}[\theta_{-n},\gamma_{-n}]U = 0.$$

- Fock space redefinition:

We apply the following tranformation for each oscillator mode (inspired from[8]) acting on the physical states.

$$\alpha^I_{-m}\ldots\alpha^J_{-n}\tilde{\alpha}^K_{-p}\ldots\tilde{\alpha}^L_{-q}|p^+,\vec{p}^T\rangle \;\rightarrow\; (U^{-1}\alpha_{-m})^I\ldots(U^{-1}\alpha_{-n})^J\left(\tilde{U}^{-1}\tilde{\alpha}_{-p}\right)^K\ldots\left(\tilde{U}^{-1}\tilde{\alpha}_{-q}\right)^L|p^+,\vec{p}^T\rangle$$

$$\left[U,\tilde{U}\right] = 0.$$

Spectrum:

Open strings

- The mass operator is symmetrized: $M^2 = \sum_{I=2}^{D-1} \sum_{n=1}^{+\infty} \frac{1}{2\alpha'} [\alpha^I_{-n}, \alpha_{nI}]_+$
- Fundamental state:Tachyonic

$$M^2 |p^+, \vec{p}^T\rangle = -\frac{1}{24\alpha'} \{Q(D-2)\} |p^+, \vec{p}^T\rangle.$$

with: D-2= $\frac{24}{Q}$.

- Massless state (Photon):

Diagonal

$$M^2\{(U^{-1}\alpha_{-1})^J |p_+ \vec{p}^T\rangle\} = \frac{1}{2\alpha'} \left\{ \begin{array}{c} (2 - \frac{Q(D-2)}{12}) - \frac{1}{\alpha'}\mu_J^{(1)} \\ -\frac{1}{\alpha'}(2\pi\alpha')^2 \nu_J^{(1)} \end{array} \right\} (U^{-1}\alpha_{-1})^J |p^+, \vec{p}^T\rangle.$$

to restaure massless state we impose:

$$\mu_J^{(1)} = -(2\pi\alpha')^2 \nu_J^{(1)}. \longrightarrow D_1 = -(2\pi\alpha')^2 T_1$$

Closed strings:

Belongs to the closed string state space.

- Closed strings states validity:

$$P = L_0^{\perp} - \tilde{L}_0^{\perp}. = N^{\perp} - \tilde{N}^{\perp}.$$

$$P_0|\lambda, \tilde{\lambda}\,\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\chi \exp\{-iP\chi\}|\lambda, \tilde{\lambda}\,\rangle = \delta_{N^{\perp}-\tilde{N}^{\perp},0}|\lambda, \tilde{\lambda}\,\rangle = |\lambda, \tilde{\lambda}\,\rangle$$

- Mass operator:

$$M^2 = \sum_{I,J=2}^{D-1}\sum_{n=1}^{+\infty}\frac{2}{\alpha'}\{\tfrac{1}{2}[\tilde{\alpha}^I_{-n}, \tilde{\alpha}_{nI}]_+ + \tfrac{1}{2}[\alpha^I_{-n}, \alpha_{nI}]_+\}.$$

- Graviton state from:

$$M^2\{(U^{-1}\alpha_{-1})^J(U^{-1}\tilde{\alpha}_{-1})^K|p_+\vec{p}^T\rangle\} = -\frac{1}{(\alpha')^2}\{\mu_J^{(-1)}+(2\pi\alpha')^2\nu_J^{(-1)}+\mu_K^{(1)}+(2\pi\alpha')^2\nu_K^{(1)}\}\{(U^{-1}\alpha_{-1})^J(U^{-1}\tilde{\alpha}_{-1})^K|p_+\vec{p}^T\rangle\}$$

- Restauring the Graviton state:

$$\mu_J^{(-1)} = -(2\pi\alpha')^2\nu_J^{(-1)}.$$

$$\mu_K^{(1)} = -(2\pi\alpha')^2\nu_K^{(1)}.$$

- Results: The spectrum is reduced, and the mass states validity would be checked again for every mass level.

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Thank you